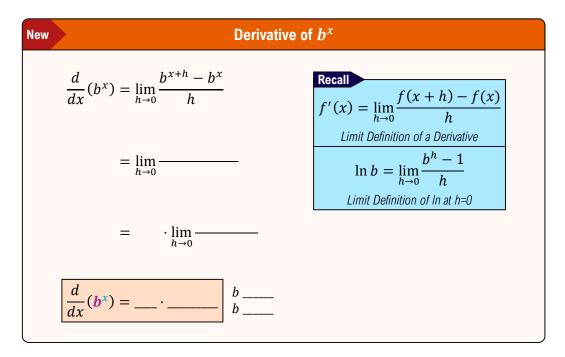
MASTER TABLE: RULES OF DIFFERENTIATION

NOTE: This table spans multiple videos.

RULES OF DIFFERENTIATION			
Name	Rule	Example	
General Exponential	$\frac{d}{dx}\mathbf{b}^{x} = \mathbf{b}^{x} \cdot \ln \mathbf{b} \qquad \qquad b > 0 \\ b \neq 1$	$\frac{d}{dx}(6^x) =$	
Natural Exponential	$\frac{d}{dx}e^x = \underline{\qquad} \cdot \ln \underline{\qquad} = \underline{\qquad}$	$\frac{d}{dx}(4e^x) =$	
General Logarithmic	$\frac{d}{dx}\log_b x = b > 0$ $b > 0$ $b \neq 1$ $x > 0$	$\frac{d}{dx}\log_8 x =$	
Natural Logarithmic	$\frac{d}{dx}\ln x = \frac{d}{dx}\log_e x = \frac{1}{x \cdot \ln \underline{\hspace{1cm}}} = x > 0$	$\frac{d}{dx}6\ln x =$	

Derivatives of General Exponential Functions

ullet We can use limits to find a derivative rule that works for all exponential functions b^x .



EXAMPLE

Find the derivative of the following functions.

$$f(x) = 6^x$$

$$g(x) = 3^{x^2 + 4x}$$

• When taking the derivative of $f(x) = b^{g(x)}$, we can apply the **chain rule** to get $f'(x) = b^{g(x)} \cdot \ln b \cdot g'(x)$.

PRACTICE

Find the derivative of the given function.

$$f(x) = 2^x - 5^x$$

(B)
$$g(x) = x^4 + 4x + 4^x$$

PRACTICE

(A)
$$y = (4x - 3x^2 + 9) \cdot 2^{5x}$$

(B)
$$h(x) = 4^{(\sqrt{x} + 3x)^{\frac{5}{4}}}$$

EXAMPLE

A medical lab uses a radioactive isotope for one of its tests. The quantity of the radioactive isotope, R, remaining in a sample after t hours is given by the function below.

$$R = 150 \left(\frac{1}{3}\right)^{\frac{t}{12}}$$

- (A) Find the instantaneous rate of change $\frac{dR}{dt}$.
- (B) Compute the instantaneous rate of change after 12 hours, 1 day, and 2 days.

(C) Interpret your results.

Derivatives of the Natural Exponential Function (e^x)

- Recall: $f(x) = e^x$ is just a special case of $f(x) = b^x$ where b = e.
 - \blacktriangleright We can use the derivative rule for general exponential functions to find the derivative of e^x .

RULES OF DIFFERENTIATION			
Name	Rule	Example	
General Exponential	$\frac{d}{dx}\boldsymbol{b}^{x} = \boldsymbol{b}^{x} \cdot \ln \boldsymbol{b} \qquad \qquad \begin{array}{c} b > 0 \\ b \neq 1 \end{array}$	$\frac{d}{dx}(6^x) = 6^x \cdot \ln 6$	
Natural Exponential	$\frac{d}{dx}e^x = \underline{\qquad} \cdot \ln \underline{\qquad} = \underline{\qquad}$	$\frac{d}{dx}(4e^x) =$	

EXAMPLE

Find the derivative of the following functions.

$$f(x) = 3e^{2x+4}$$

$$g(x) = xe^{5x}$$

• When taking the derivative of $f(x) = e^{g(x)}$, we can apply the **chain rule** to get $f'(x) = e^{g(x)} \cdot g'(x)$.

PRACTICE

Find the derivative of the given function.

(A)
$$f(x) = -3e^x + 5x - 2$$

$$(B) g(x) = 7e^x + 2x^3$$

EXAMPLE

$$y = e^{\sqrt{x^3 - 2x}}$$

PRACTICE

$$(A) y = x^2 e^{3x^2 + 5x}$$

$$(B)$$

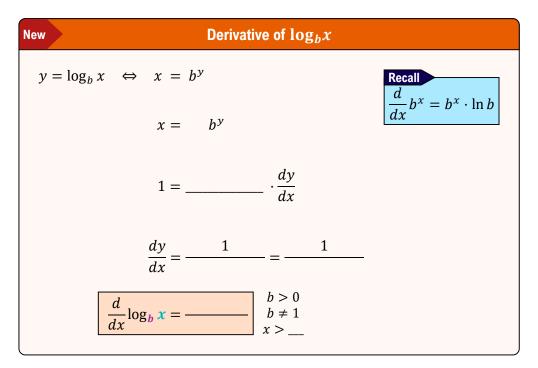
$$f(t) = \frac{e^{3t}}{t - 2e^{-t}}$$

EXAMPLE

Find an equation of the tangent line to the function $y = 3e^x - x$ at x = 0.

Derivatives of General Logarithmic Functions

♦ Since $y = \log_b x$ is equivalent to _____, we can differentiate _____ sides of that equation to find $\frac{d}{dx}\log_b x$.



EXAMPLE

Find the derivative of the following functions.

$$(A) g(x) = \log_8 x$$

$$f(x) = \log_5(x^2)$$

• When taking the derivative of $f(x) = \log_b(g(x))$, we can apply the **chain rule** to get $f'(x) = \frac{1}{g(x) \cdot \ln b} \cdot g'(x)$.

PRACTICE

Find the derivative of the given function.

$$f(x) = 8\log_2 x$$

$$(B) g(t) = 2t + \log_5 t$$

EXAMPLE

Find the derivative of $y = \log_4 \sqrt{\frac{x}{4x+1}}$.

PRACTICE

$$f(x) = (x^3 + 2x) \cdot \log_5 x$$

(B)
$$g(t) = \log_5(7^{t^2+4})$$

Derivatives of the Natural Logarithmic Function

- ♦ Recall: $f(x) = \ln x$ is just a special case of $f(x) = \log_b x$ where b = e.
 - ▶ We can use the derivative rule for general log functions to find the derivative of $f(x) = \ln x = \log x$.

RULES OF DIFFERENTIATION			
Name	Rule	Example	
General Logarithmic	$\frac{d}{dx}\log_b x = \frac{1}{x \cdot \ln b} \qquad b > 0$ $b \neq 1$ $x > 0$	$\frac{d}{dx}\log_8 x = \frac{1}{8\ln 8}$	
Natural Logarithmic	$\frac{d}{dx}\ln x = \frac{d}{dx}\log_e x = \frac{1}{x \cdot \ln \underline{\hspace{1cm}}} = x > 0$	$\frac{d}{dx}6\ln x =$	

EXAMPLE

Find the derivative of the given function.

$$f(x) = \ln(x^2 + 4x)$$

$$g(x) = x \ln x^3$$

• When taking the derivative of $f(x) = \ln(g(x))$, we can apply the **chain rule** to get $f'(x) = \frac{1}{g(x)} \cdot g'(x)$.

PRACTICE

Find the derivative of the given function.

$$f(x) = 2x^3 - 1 + \ln x$$

$$g(x) = e^x + \ln x^5$$

PRACTICE

(A)
$$h(x) = \ln\left(\frac{\sqrt{x+1}}{x^2+3}\right)$$

$$(B) y = x^2 \ln(x^2)$$

$$g(x) = e^{x^2 \ln x}$$

EXAMPLE

The time t, in days that it takes the number of bacteria in a sample to reach B is given by the function below. Find t'(1000) and explain what it represents.

$$t(B) = 42 \ln \left(\frac{650}{1500 - B} \right)$$