MASTER TABLE: RULES OF DIFFERENTIATION

NOTE: This table spans multiple videos.

RULES OF DIFFERENTIATION			
Name	**Rule**		**Example**
General Exponential	$\frac{d}{dx} b^x = b^x \cdot \ln b$	$b > 0$ $b \neq 1$	$\frac{d}{dx}(6^x) =$
Natural Exponential	$\frac{d}{dx} e^x =$ ___ $\cdot \ln$ ___ $=$ _______		$\frac{d}{dx}(4e^x) =$
General Logarithmic	$\frac{d}{dx} \log_b x =$ ________	$b > 0$ $b \neq 1$ $x > 0$	$\frac{d}{dx} \log_8 x =$
Natural Logarithmic	$\frac{d}{dx} \ln x = \frac{d}{dx} \log_e x = \frac{1}{x \cdot \ln \_\_\_\_\_} =$	$x > 0$	$\frac{d}{dx} 6 \ln x =$

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

Derivatives of General Exponential Functions

◆ We can use limits to find a derivative rule that works for *all* exponential functions b^x.

New — **Derivative of b^x**

$$\frac{d}{dx}(b^x) = \lim_{h \to 0} \frac{b^{x+h} - b^x}{h}$$

$$= \lim_{h \to 0} \underline{\hspace{3cm}}$$

$$= \quad \cdot \lim_{h \to 0} \underline{\hspace{2.5cm}}$$

$$\frac{d}{dx}(b^x) = \underline{\quad} \cdot \underline{\hspace{2cm}}$$

b ____

b ____

Recall

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Limit Definition of a Derivative

$$\ln b = \lim_{h \to 0} \frac{b^h - 1}{h}$$

Limit Definition of ln at h=0

EXAMPLE Find the derivative of the following functions.

(A) $f(x) = 6^x$

(B) $g(x) = 3^{x^2+4x}$

◆ When taking the derivative of $f(x) = b^{g(x)}$, we can apply the **chain rule** to get $f'(x) = b^{g(x)} \cdot \ln b \cdot g'(x)$.

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

PRACTICE Find the derivative of the given function.

(A)

$f(x) = 2^x - 5^x$

(B)

$g(x) = x^4 + 4x + 4^x$

PRACTICE Find the derivative of the given function.

(A)

$y = (4x - 3x^2 + 9) \cdot 2^{5x}$

(B)

$h(x) = 4^{(\sqrt{x}+3x)^{\frac{5}{4}}}$

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

EXAMPLE

A medical lab uses a radioactive isotope for one of its tests. The quantity of the radioactive isotope, R, remaining in a sample after t hours is given by the function below.

$$R = 150\left(\frac{1}{3}\right)^{\frac{t}{12}}$$

$(\boldsymbol{A})$ Find the instantaneous rate of change $\frac{dR}{dt}$.

$(\boldsymbol{B})$ Compute the instantaneous rate of change after 12 hours, 1 day, and 2 days.

$(\boldsymbol{C})$ Interpret your results.

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

Derivatives of the Natural Exponential Function (e^x)

◆ Recall: $f(x) = e^x$ is just a special case of $f(x) = b^x$ where $b = e$.

► We can use the derivative rule for general exponential functions to find the derivative of e^x.

RULES OF DIFFERENTIATION		
Name	**Rule**	**Example**
General Exponential	$\frac{d}{dx} b^x = b^x \cdot \ln b$ $\quad b > 0$, $b \neq 1$	$\frac{d}{dx}(6^x) = 6^x \cdot \ln 6$
Natural Exponential	$\frac{d}{dx} e^x = \_\_\_ \cdot \ln \_\_\_ = \_\_\_\_\_$	$\frac{d}{dx}(4e^x) =$

EXAMPLE Find the derivative of the following functions.

(A) $f(x) = 3e^{2x+4}$

(B) $g(x) = xe^{5x}$

◆ When taking the derivative of $f(x) = e^{g(x)}$, we can apply the **chain rule** to get $f'(x) = e^{g(x)} \cdot g'(x)$.

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

PRACTICE Find the derivative of the given function.

$(\boldsymbol{A})$

$f(x) = -3e^x + 5x - 2$

$(\boldsymbol{B})$

$g(x) = 7e^x + 2x^3$

EXAMPLE Find the derivative of the given function.

$y = e^{\sqrt{x^3 - 2x}}$

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

PRACTICE Find the derivative of the given function.

$(\boldsymbol{A})$

$$y = x^2 e^{3x^2+5x}$$

$(\boldsymbol{B})$

$$f(t) = \frac{e^{3t}}{t - 2e^{-t}}$$

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

EXAMPLE Find an equation of the tangent line to the function $y = 3e^x - x$ at $x = 0$.

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

Derivatives of General Logarithmic Functions

- Since $y = \log_b x$ is equivalent to __________, we can differentiate __________ sides of that equation to find $\frac{d}{dx}\log_b x$.

New | **Derivative of $\log_b x$**

$y = \log_b x \quad \Leftrightarrow \quad x = b^y$

Recall
$\frac{d}{dx}b^x = b^x \cdot \ln b$

$x = \quad b^y$

$1 = \_\_\_\_\_\_\_\_\_\_ \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\_\_\_\_\_\_\_\_\_\_} = \frac{1}{\_\_\_\_\_\_\_\_\_\_}$

$\frac{d}{dx}\log_b x = \_\_\_\_\_\_\_\_\_\_$

$b > 0$
$b \neq 1$
$x > \_\_$

EXAMPLE Find the derivative of the following functions.

(A) $g(x) = \log_8 x$

(B) $f(x) = \log_5(x^2)$

- When taking the derivative of $f(x) = \log_b(g(x))$, we can apply the **chain rule** to get $f'(x) = \frac{1}{g(x) \cdot \ln b} \cdot g'(x)$.

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

PRACTICE Find the derivative of the given function.

(A)

$f(x) = 8\log_2 x$

(B)

$g(t) = 2t + \log_5 t$

EXAMPLE Find the derivative of $y = \log_4 \sqrt{\frac{x}{4x+1}}$.

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

PRACTICE Find the derivative of the given function.

(***A***)

$f(x) = (x^3 + 2x) \cdot \log_5 x$

(***B***)

$g(t) = \log_5(7^{t^2+4})$

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

Derivatives of the Natural Logarithmic Function

◆ Recall: $f(x) = \ln x$ is just a special case of $f(x) = \log_b x$ where $b = e$.

► We can use the derivative rule for general log functions to find the derivative of $f(x) = \ln x = \log\_\_\, x$.

RULES OF DIFFERENTIATION			
Name	**Rule**		**Example**
General Logarithmic	$\frac{d}{dx}\log_b x = \frac{1}{x \cdot \ln b}$	$b > 0$ $b \neq 1$ $x > 0$	$\frac{d}{dx}\log_8 x = \frac{1}{8 \ln 8}$
Natural Logarithmic	$\frac{d}{dx}\ln x = \frac{d}{dx}\log_e x = \frac{1}{x \cdot \ln \_\_\_\_}$ =	$x > 0$	$\frac{d}{dx} 6 \ln x =$

EXAMPLE Find the derivative of the given function.

(A) $f(x) = \ln(x^2 + 4x)$

(B) $g(x) = x \ln x^3$

◆ When taking the derivative of $f(x) = \ln(g(x))$, we can apply the **chain rule** to get $f'(x) = \frac{1}{g(x)} \cdot g'(x)$.

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

PRACTICE Find the derivative of the given function.

(A)

$$f(x) = 2x^3 - 1 + \ln x$$

(B)

$$g(x) = e^x + \ln x^5$$

PRACTICE Find the derivative of the given function.

(A)

$$h(x) = \ln\left(\frac{\sqrt{x+1}}{x^2+3}\right)$$

(B)

$$y = x^2 \ln(x^2)$$

(C)

$$g(x) = e^{x^2 \ln x}$$

EXAMPLE

The time t, in days that it takes the number of bacteria in a sample to reach B is given by the function below. Find $t'(1000)$ and explain what it represents.

$$t(B) = 42 \ln\left(\frac{650}{1500 - B}\right)$$