

TOPIC: THE FUNDAMENTAL THEOREM OF CALCULUS

Fundamental Theorem of Calculus (FTC) Part 1

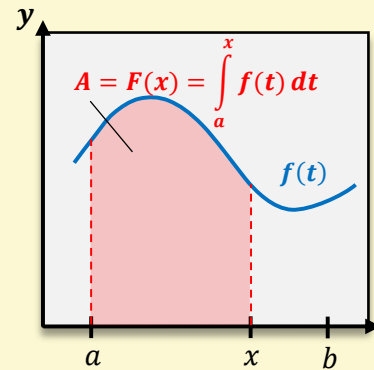
◆ Recall: $F(x)$ is an antiderivative of $f(x)$.

► Fundamental Theorem of Calculus Part 1 connects derivatives to definite integrals.

The Fundamental Theorem of Calculus Part 1

If $f(x)$ is continuous on $[a, b]$, and $F(x) = \int_a^x f(t) dt$, then:

$$F'(x) = \frac{d}{dx} \int_a^x f(t) dt = \underline{\hspace{2cm}}$$



◆ If upper bound is a *fcn* of x , use FTC w/ _____ rule to find the derivative.

New

$$\frac{d}{dx} \int_a^{g(x)} f(t) dt = f(g(x)) \cdot g'(x)$$

EXAMPLE

Use the Fundamental Theorem to find $\frac{dy}{dx}$ for the following.

(A)

$$y = \int_1^x (t^3 - 3t^2 + 4) dt$$

(B)

$$y = \int_5^{x^2} \frac{2}{1+t} dt$$

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PRACTICE

Given the definite integral $F(x) = \int_3^x [t^8 - \sin(t^4)] dt$, find the derivative $F'(x)$.

PRACTICE

Given the definite integral $F(x) = \int_{12}^{20x} \left(h^4 + \frac{63h}{\sqrt{h^5}} \right) dh$, find the derivative $F'(x)$.

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EXAMPLE

Find the derivative $F'(x)$ of the following definite integrals.

(A)

$$F(x) = \int_0^x \left(t^4 - 30t^3 + \frac{1}{3}t^2 + 30t - 1000 \right) dt$$

(B)

$$F(x) = \int_0^{x^2-13x} \frac{(h^8)}{\sqrt{h} - \sin(h)} dh$$

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Fundamental Theorem of Calculus (FTC) Part 2

- ◆ Recall: The Fundamental Theorem Part 1 gives a relationship between definite integrals and antiderivatives.
- ▶ FTC Part 2 allows us to evaluate a definite integral using the antiderivative at the upper and lower bounds.

The Fundamental Theorem of Calculus Part 2

If $f(x)$ is continuous on $[a, b]$, and $F(x)$ is *any* antiderivative of $f(x)$ on $[a, b]$, then:

$$\int_a^b f(x) dx = \text{_____} - \text{_____}$$

EXAMPLE

Evaluate the following integrals.

(A)

$$y = \int_2^5 2x dx$$

(B)

$$y = \int_1^4 (x^2 - 4x + 5) dx$$

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PRACTICE

Evaluate the following integrals:

(A)

$$\int_0^3 4dx$$

(B)

$$\int_1^4 (2x + 1) dx$$

(C)

$$\int_{-1}^2 (x^2 - 3x + 2) dx$$

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PRACTICE

Evaluate the following integrals:

(D)

$$\int_0^1 (2x^3 - x^2 + 4x) dx$$

(E)

$$\int_2^3 x^{\frac{5}{2}} dx$$

(F)

$$\int_1^2 \frac{5}{x^2} dx$$