

TOPIC: INTRODUCTION TO VOLUME & THE DISK METHOD

Introduction to Cross Sections



- ◆ Recall: To find area, add the area of many small rectangles. To find volume, add the volume of many thin slices.
 - Volume of slice = Area of Cross Section • width. A **cross section** is a 2D shape we get from cutting a 3D solid.

EXAMPLE

Set up an int. for the volume of a solid with base $f(x) = 4 - x^2$ on $[0, 2]$ with square cross sections.

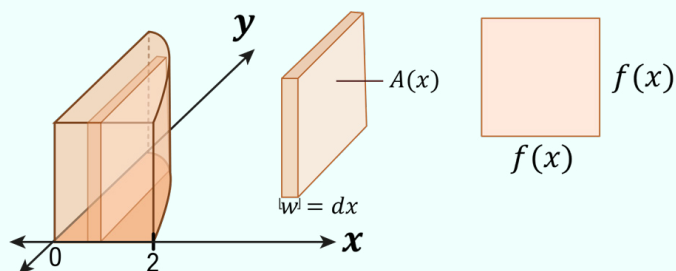
Recall	Integrals for Area	New	Integrals for Volume
	<p>A graph of the function $f(x) = 4 - x^2$ on the interval $[0, 2]$. The area under the curve is approximated by several vertical blue rectangles. A single rectangle is highlighted with height $h = f(x)$ and width $w = dx$. The x-axis is labeled from 0 to 2, and the y-axis from 0 to 5.</p> <div>$Area = \int_a^b f(x) dx$</div>		<p>A 3D diagram showing a solid with a base defined by $f(x) = 4 - x^2$ on $[0, 2]$. The solid is composed of thin slices with square cross sections. A single slice is shown with side length $A(x)$ and thickness dx. A 2D square cross-section is also shown with side length $f(x)$.</p> <div>$Volume = \int_a^b A(x) dx$</div>

- ◆ ALWAYS sketch the solid and a front facing cross section.

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EXAMPLE

Find the volume of a solid with base $f(x) = 4 - x^2$ on $[0, 2]$, with square cross sections.



$$\text{Volume} = \int_0^2 (4 - x^2)^2 dx$$

Recall

$$\text{Volume} = \int_a^b A(x) dx$$

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Finding Volume Using Disks

◆ Recall: To find volume of a 3D solid, integrate the area function of a cross section.

Recall

$$Volume = \int_a^b A(x) dx$$

► A **Solid of Revolution** is a solid formed by revolving a curve around an axis (x -axis, y -axis, $y = 2$, etc.).

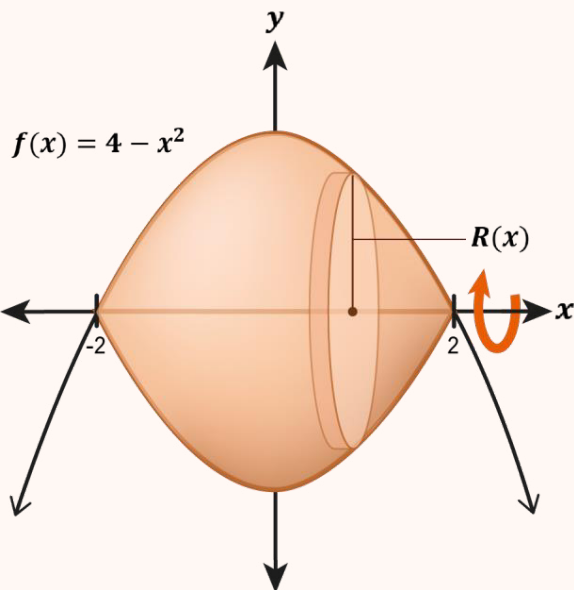
► The cross sections of a solid of rev. are _____ (*disks*) whose radius = the dist. from the _____ to the _____.

EXAMPLE

Set up an int. for the volume of a solid formed by rotating $f(x) = 4 - x^2$ on $[-2, 2]$ about the x -axis.

New

Solids of Revolution: x — Axis

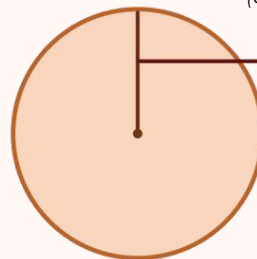


Recall

$$Area \text{ of circle} = \pi r^2$$

$$Volume = \int_a^b \underline{\hspace{2cm}} dx$$

(Solid of Revolution – Disk Method)



$$R(x) = \text{curve} - \text{axis}$$

$$= \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

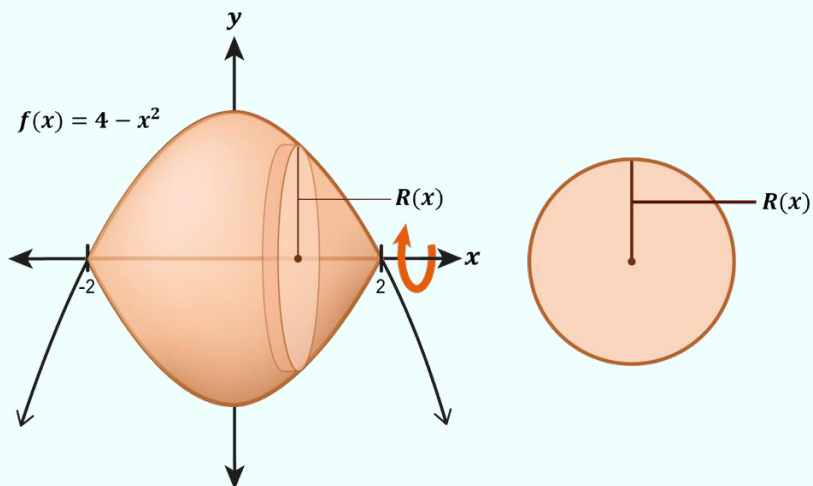
$$= \underline{\hspace{2cm}}$$

$$Volume =$$

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EXAMPLE

Find the volume of the solid formed by rotating $f(x) = 4 - x^2$ on $[-2, 2]$ about the x -axis.



$$Volume = \int_{-2}^2 \pi(4 - x^2)^2 dx$$

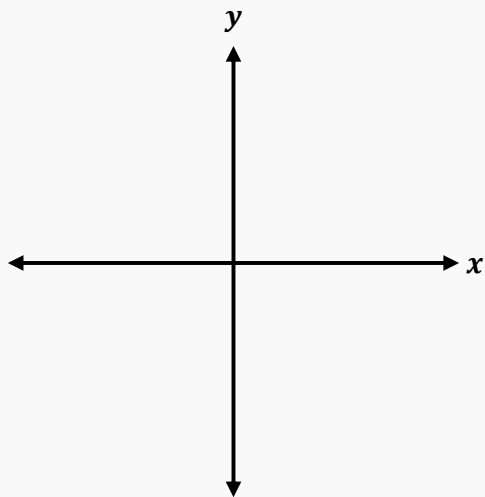
Recall

$$Volume = \int_a^b \pi[R(x)]^2 dx$$

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PRACTICE

Find the volume of the solid obtained by rotating the region bounded by $y = x + 4$, $y = 0$, $x = 1$ & $x = 5$ about the x -axis.



Recall

$$Volume = \int_a^b \pi [R(x)]^2 dx$$