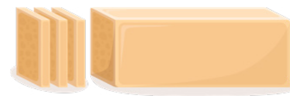
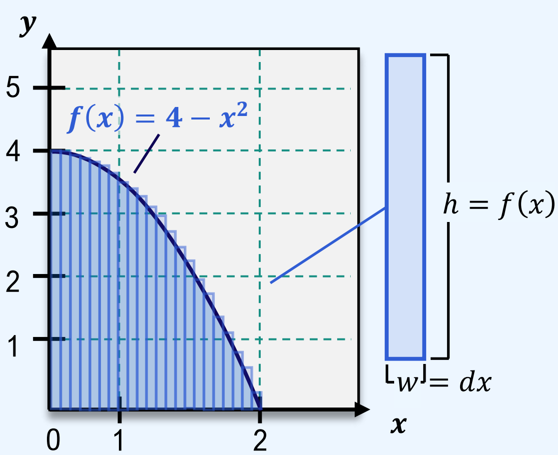
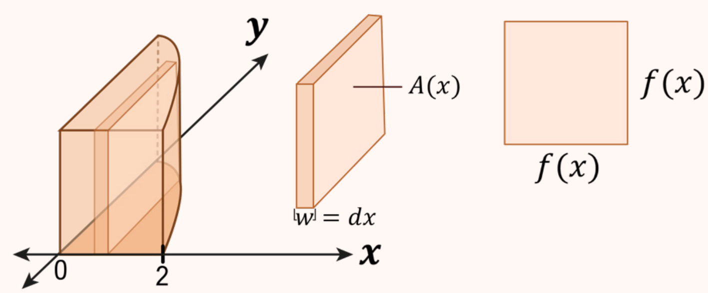
TOPIC: INTRODUCTION TO VOLUME & THE DISK METHOD

Introduction to Cross Sections



- Recall: To find area, add the area of many small rectangles. To find volume, add the volume of many thin slices.
  - Volume of slice = Area of Cross Section • width. A **cross section** is a 2D shape we get from cutting a 3D solid.

EXAMPLE Set up an int. for the volume of a solid with base $f(x) = 4 - x^2$ on $[0, 2]$ with square cross sections.

Recall **Integrals for Area**	New **Integrals for Volume**
	
$Area = \int_a^b f(x)dx$	$Volume = \int_a^b A(x)dx$

- *ALWAYS* sketch the solid and a front facing cross section.

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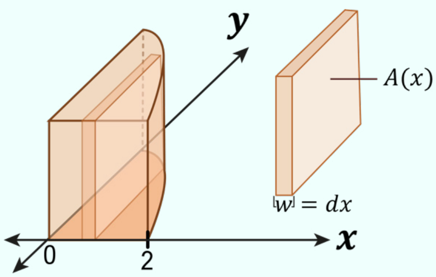
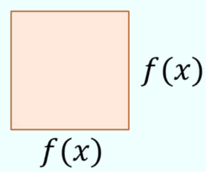
EXAMPLE Find the volume of a solid with base $f(x) = 4 - x^2$ on $[0, 2]$, with square cross sections.





Recall

$$Volume = \int_a^b A(x)dx$$

$$Volume = \int_0^2 (4 - x^2)^2 \, dx$$

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Finding Volume Using Disks

◆ Recall: To find volume of a 3D solid, integrate the area function of a cross section.

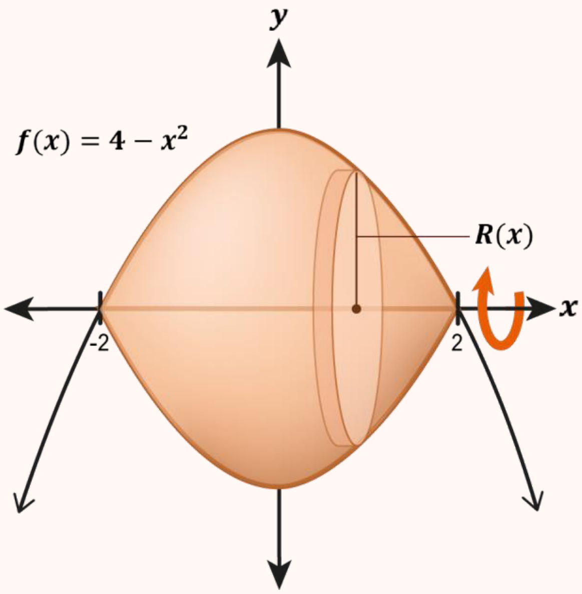
Recall $Volume = \int_a^b A(x)dx$

► A **Solid of Revolution** is a solid formed by revolving a curve around an axis (x-axis, y-axis, $y = 2$, etc.).

► The cross sections of a solid of rev. are __________ (*disks*) whose radius = the dist. from the ________ to the _____.

EXAMPLE Set up an int. for the volume of a solid formed by rotating $f(x) = 4 - x^2$ on $[-2,2]$ about the x-axis.

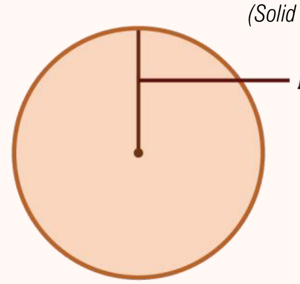
New **Solids of Revolution: x — Axis**



Recall $Area\ of\ circle = \pi r^2$

$Volume = \int_a^b$ __________ dx

(Solid of Revolution – Disk Method)



$R(x)$ = curve – axis

= ___________

= ___________

= ___________

$Volume =$

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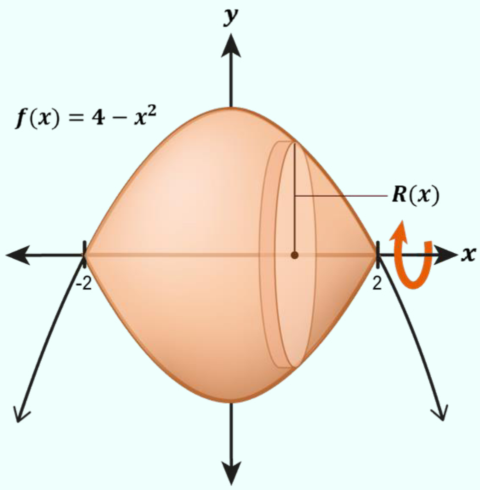
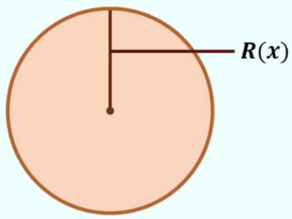
EXAMPLE Find the volume of the solid formed by rotating $f(x) = 4 - x^2$ on $[-2,2]$ about the x-axis.





Recall

$$Volume = \int_a^b \pi[R(x)]^2 dx$$

$$Volume = \int_{-2}^{2} \pi(4 - x^2)^2 dx$$

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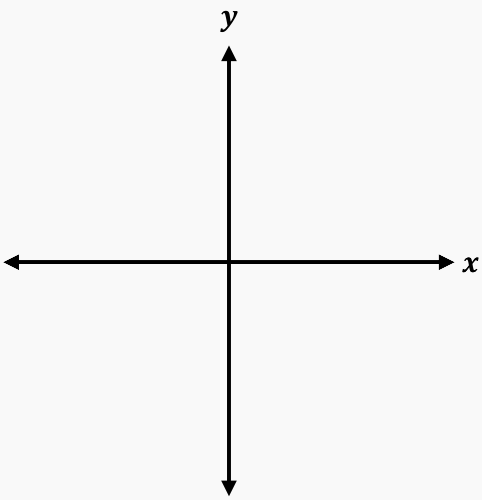
PRACTICE Find the volume of the solid obtained by rotating the region bounded by $y = x + 4$, $y = 0$, $x = 1$ & $x = 5$ about the x-axis.



Recall

$$Volume = \int_a^b \pi[R(x)]^2 dx$$