Solving Exponential Equations Using Like Bases

ullet We **evaluated** exp. functions for given x; we **solve** exp. *equations* for unknown to make the statement _____.

► To solve exp. equations, rewrite each side to have same _____ & set _____ equal, solving for ____.

Recall	Exponential Function	New	Exponential <i>Equation</i>	
	$f(x)=2^x$		$16=2^x$	
	$f(4)=2^4$		$2=2^x$	
	f(4) = 16		=	

EXAMPLE

Solve each exponential equation by expressing each side as a power of the same base.

(A)
$$64 = 2^x$$
 $| \textbf{(B)} |$ $5^{x+1} = \sqrt{5}$ $| \textbf{(C)} |$ $27 = 9^x$

COMMON POWERS		
Squares	Cubes	
$2^{2} = 4$ $3^{2} = 9$ $4^{2} = 16$ $5^{2} = 25$ $6^{2} = 36$	$2^{3} = 8$ $3^{3} = 27$ $4^{3} = 64$ $5^{3} = 125$	
$7^2 = 49$	Other	
$8^2 = 64$ $9^2 = 81$ $10^2 = 100$	$2^4 = 16$ $2^5 = 32$ $3^4 = 81$	

PRACTICE

Solve the exponential equations.

(**A**)
$$4^{x+7} = 16$$

$$100^x = 10^{x+17}$$

(*C*)
$$81^{x+1} = 27^{x+5}$$

Solving Exponential Equations Using Logs

◆ If you **CANNOT** rewrite sides of exponential equation to have the same base, solve using log or ln.

Like Bases

Unlike Bases $16 = 2^x$ $2^4 = 2^x$ 4 = xUnlike Bases $2^{x} = 2^x$ $2^{x} = 2^x$

Properties of Logarithms			
Name	Property		
Product Rule	$\log_b(m \times n) = \log_b m + \log_b n$		
Quotient Rule	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$		
Power Rule	$\log_b m^n = n \log_b m$		

EXAMPLE

Solve each exponential equation by using logs.

$$(A) 10^x + 64 = 100$$

 (\mathbf{B}) $3 = 2^{x+1}$

HOW TO: SOLVE EXP. EQNS WITH LOGS

- 1) _____ exp. expression(s)
- 2) If base 10: Take ____ of both sides
 If NOT base 10: Take ___ of both sides
- 3) Use $\log \text{ rules to get } x \text{ out of exponent}$
- 4) _____ for *x*
- 5) (If asked) Approximate using calculator

◆ Recall: The log or In of some number is a _____, not a variable! Treat it like any other number.

PRACTICE

Solve the exponential equations.

 (\boldsymbol{A})

$$2 \cdot 10^{3x} = 5000$$

(**B**)

$$900 = 10^{x+17}$$

(**C**)

$$e^{2x+5} = 8$$

(D)

$$7^{2x^2 - 8} = 1$$

Solving Logarithmic Equations

- When solving log equations, you'll encounter two possibilities: $\log_b M = \log_b N$ **OR** $\log_b M = c$
 - If a logarithmic equation can be written as $\log_b M = \log_b N$, set ____ = ___ & solve for x.

Recall Exponential Equation	New Logarithmic Equation
$16=2^x$	$\log_2(x+1) = \log_2 5$
$2^4 = 2^x$	=
4 = x	=

EXAMPLE

Solve the log equation.

$$\ln(x+4) - \ln 2 = \ln 8$$

Properties of Logarithms				
Name	Property			
Product Rule	$\log_b(m \times n) = \log_b m + \log_b n$			
Quotient Rule	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$			
Power Rule	$\log_b m^n = n \log_b m$			

▶ If a log equation **CANNOT** be written as $\log_b M = \log_b N$ put in _____ form, then solve for x.

Recall:

Equals
$$C = \log_h M$$

To the power of

EXAMPLE

Solve the log equation.

$$\log_2(4x) = 5$$

HOW TO: SOLVE LOG EQNS: EXP FORM

- 1) _____log expression
- 2) Put in exponential form
- 3) _____ for x.
- 4) _____ solution by plugging x in M
 - If M > 0, DONE
 - If **M** < **0**, ____ a solution

PRACTICE

Solve the logarithmic equations.

(A)
$$\log_3(3x+9) = \log_3 5 + \log_3 12$$

$$\log(x+2) + \log 2 = 3$$

(*C*)
$$\log_7(6x + 13) = 2$$