TOPIC: INTEGRATION BY PARTS

Topic Resource: Basic Integration Rules

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Intro to Integration by Parts

◆ To integrate the product of two functions, if all other methods fail, use **integration by parts (IBP)**.

► To get the IBP formula, we have to __________ the derivative product rule.

New **Integration by Parts**

Recall

$$\boxed{\frac{d}{dx}[f(x)\cdot g(x)] = f'(x)\cdot g(x) + f(x)\cdot g'(x)}$$

$$\frac{d}{dx}[f(x)\cdot g(x)] \quad = \quad f'(x)\cdot g(x) \quad + \quad f(x)\cdot g'(x)$$

$$\_\_\_\_\_\_\cdot\_\_\_\_\_\_ = \int \_\_\_\_\_\_\cdot\_\_\_\_\_\_\,dx + \int \_\_\_\_\_\_\cdot\_\_\_\_\_\_\,dx$$

$$\int \underbrace{\_\_\_\_\_\_}\cdot\underbrace{\_\_\_\_\_\_}\,dx = \underbrace{\_\_\_\_\_\_}\cdot\underbrace{\_\_\_\_\_\_} - \int \underbrace{\_\_\_\_\_\_}\cdot\underbrace{\_\_\_\_\_\_}\,dx$$

$$\boxed{\int \boldsymbol{u}\cdot \boldsymbol{dv} = \_\_\_\cdot\_\_\_ - \int \_\_\_\cdot\_\_\_}$$

◆ When using IBP, $\boldsymbol{u}$ should become simpler when *differentiated* and $\boldsymbol{dv}$ should be a function that's easily *integrated*.

EXAMPLE Find the indefinite integral using integration by parts.

$$\int 6xe^x\,dx$$

$\boldsymbol{u} = \_\_\_\_\_\_$ $\qquad \boldsymbol{dv} = \_\_\_\_\_\_\,dx$

$\boldsymbol{du} = \_\_\_\_\_\,dx$ $\qquad \boldsymbol{v} = \_\_\_\_\_\_$

◆ $\boldsymbol{u}$ will *often* be the function that appears FIRST here: **L**og, **I**nverse, **A**lgebraic, **T**rig, **E**xponential **LIATE**

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EXAMPLE Find the integral.

$$\int \ln x \, dx$$

Recall

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

LIATE

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PRACTICE Find the integral.

$(\boldsymbol{A})$

$$\int s\, e^{3s} ds$$

$(\boldsymbol{B})$

$$\int x^2 \ln x \ \ dx$$

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Repeated Integration by Parts

◆ Integration by parts may need to be _______________ to evaluate an integral.

EXAMPLE Find the indefinite integral $\int 3x^2e^x\,dx$.

Recall

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

$$\int (3x^2e^x)\,dx = \_\_\_\_\_ \cdot \_\_\_\_\_ - \int (\_\_\_\_\_ \cdot \_\_\_\_\_)$$

$u =$ _____ $dv =$ _____

$du =$ _____ $v =$ _____

$$= \_\_\_\_\_ \cdot \_\_\_\_\_ - \left[\_\_\_\_\_ \cdot \_\_\_\_\_ - \int (\_\_\_\_\_ \cdot \_\_\_\_\_)\right]$$

$u =$ _____ $dv =$ _____

$du =$ _____ $v =$ _____

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EXAMPLE Find the indefinite integral.

(*A*)

$$\int (t^2 + 2t - 3)\, e^{4t}\, dt$$

(*B*)

$$\int (\ln x)^2\, dx$$

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PRACTICE Find the indefinite integral.

$$\int r^2 e^{-r} dr$$

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Tabular Integration by Parts

◆ For more complicated integrals that require repeated IBP, it can be helpful to organize into a ____________________.

EXAMPLE Find the integral $\int 3x^2e^{2x}\,dx$.

Recall **Integration by Parts**

$$\boxed{\int u\cdot dv = u\cdot v - \int v\cdot du}$$

$$\int \underbrace{(3x^2}_{u}\underbrace{e^{2x})}_{dv}\,dx$$

$$= \underbrace{3x^2}_{u}\underbrace{\bigg(\frac{1}{2}e^{2x}\bigg)}_{v} - \int \bigg(\underbrace{\frac{1}{2}e^{2x}}_{v}\cdot\underbrace{6x\bigg)\,}_{du}dx$$

$$= \underbrace{3x^2}_{u}\cdot\underbrace{\frac{1}{2}e^{2x}}_{v} - \bigg(\underbrace{6x}_{u}\cdot\underbrace{\frac{1}{4}e^{2x}}_{v} - \int \underbrace{\frac{1}{4}e^{2x}}_{v}\cdot\underbrace{6\,dx}_{du}\bigg)$$

$$= \frac{3}{2}x^2e^{2x} - \frac{3}{2}xe^{2x} + \frac{3}{4}e^{2x} + C$$

New **Tabular Integration by Parts**

u & its **D**erivatives

dv & its antiderivatives (**I**ntegrals)

	Alternating Signs	D	I
Start with:	$[+\mid-]$		
	$[+\mid-]$		
	$[+\mid-]$		
	$[+\mid-]$		

$$\int (3x^2e^{2x})\,dx = (\_\_\_\_\_)(\_\_\_\_\_\_)\qquad(\_\_\_\_\_)(\_\_\_\_\_\_)$$

$$(\_\_\_\_\_)(\_\_\_\_\_)\qquad\int(\_\_\_\_\_)(\_\_\_\_\_) + C$$

$$=$$

◆ In the D column, differentiate u to ______ if possible or until the product of the bottom row is a basic integral.

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EXAMPLE Evaluate the indefinite integral using the tabular method.

$$\int t^3 \ln t \ dt$$

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PRACTICE Evaluate the indefinite integral.

$(\boldsymbol{A})$

$$\int (s^2 + 4)e^{3s}\,ds$$

$(\boldsymbol{B})$

$$\int \frac{\ln x}{x^4}\,dx$$

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Integration by Parts: Definite Integrals

- To find a definite integral using IBP, solve like an indefinite integral, then apply bounds to __________ parts at the end.

EXAMPLE Evaluate the definite integral using integration by parts.

$$\int_{0}^{2} 6xe^{x}\,dx$$

New

$$\int_{\_\_}^{\_\_} u \cdot dv = u \cdot v \Big|_{\_\_}^{\_\_} - \int_{\_\_}^{\_\_} v \cdot du$$

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PRACTICE Evaluate the definite integral.

$$\int_1^2 x \ln x \; dx$$

Recall

$$\int_a^b u \cdot dv = u \cdot v \Big|_a^b - \int_a^b v \cdot \, du$$

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PRACTICE

The rate of growth of a particular bacteria is given by $b'(t) = 4t^2e^{.5t}$ where t is time in days. What is the total growth of the population of bacteria during the first 5 days?

Recall

$$\int_a^b u \cdot dv = u \cdot v \Big|_a^b - \int_a^b v \cdot du$$