

TOPIC: CURVE SKETCHING

Summary of Curve Sketching

◆ We can use f' & f'' along with basic graphing knowledge to sketch the graph of f .

EXAMPLE

Sketch the graph of $f(x) = x^3 - 3x^2 + 4$.

HOW TO: Sketch Graph of f Using f' & f''

1) Determine **domain** of f : _____

2) Find x & y **intercepts**:

$$f(x) = 0 \quad f(0) = 0^3 - 3(0)^2 + 4$$

$$(x - 2)^2(x + 1) = 0$$

3) If needed, find **asymptotes**

4) **Symmetry** $\rightarrow f(-x) = \begin{bmatrix} f(x) & -f(x) \\ \text{sym. about} & \text{y-axis} \mid \text{origin} \end{bmatrix}$

$$(-x)^3 - 3(-x)^2 + 4 =$$

5) a. **Inc/Dec** $\rightarrow$ Find crit. pts: $f'(x) = 0$ / DNE

$$f'(x) = 3x^2 - 6x = 3x(x - 2) \rightarrow$$

f'

←————→ x

f''

b. **Concave up/down** $\rightarrow$ Inf. pts: $f''(x) = 0$ / DNE

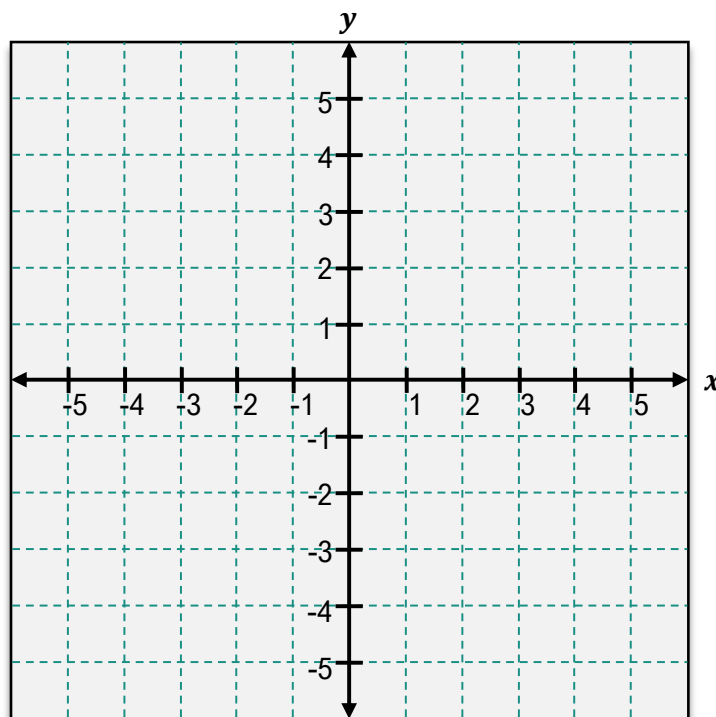
$$f''(x) = 6x - 6 = 6(x - 1) \rightarrow$$

6) **Local Extrema** $\rightarrow$ Use (5a): $+$ to $-$, local **MAX**

$f(\text{---}) = \text{---}$ $-$ to $+$, local **MIN**

$f(\text{---}) = \text{---}$

7) **Connect** points with smooth curve based on 4-6



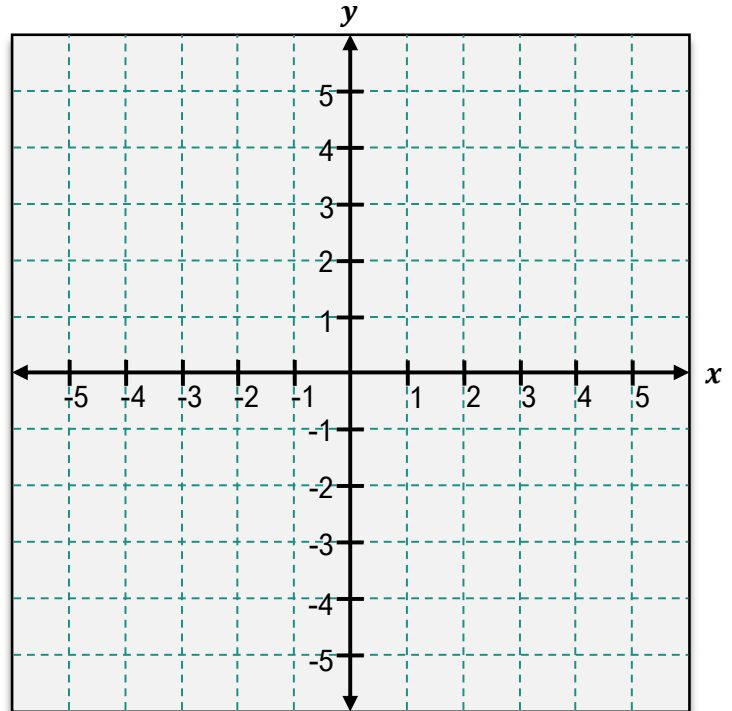
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EXAMPLE

Sketch the graph of $f(x) = \frac{x-1}{x+2}$.

HOW TO: Sketch Graph of f Using f' & f''

- 1) Determine **domain** of f : _____
- 2) Find x & y **intercepts**: $f(x) = 0$ & $f(0)$
- 3) If needed, find **asymptotes**
- 4) **Symmetry** $\rightarrow f(-x) = \begin{bmatrix} f(x) & | & -f(x) \end{bmatrix}$
sym. about $\begin{bmatrix} y\text{-axis} & | & \text{origin} \end{bmatrix}$
- 5) a. **Inc/Dec** $\rightarrow$ Find crit. pts: $f'(x) = 0$ / DNE
 f'
 $\longleftrightarrow x$
 f''
b. **Concave up/down** $\rightarrow$ Inf. pts: $f''(x) = 0$ / DNE
- 6) **Local Extrema** $\rightarrow$ Use (5a): $+$ to $-$, local **MAX**
 $-$ to $+$, local **MIN**
- 7) **Connect** points with smooth curve based on 4-6



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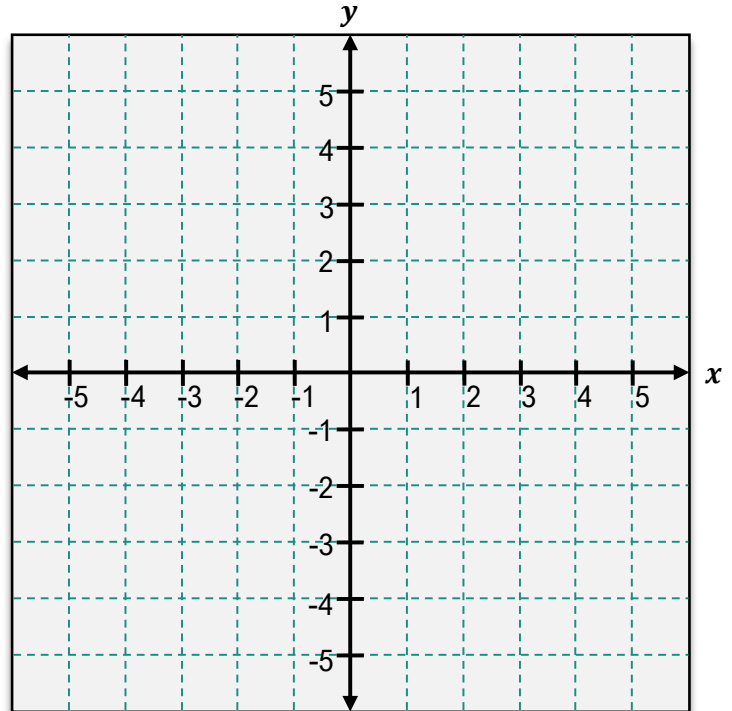
EXAMPLE

Sketch the graph of $g(x) = \sqrt{x^3 + 8}$.

HOW TO: Sketch Graph of f Using f' & f''

- 1) Determine **domain** of f : _____
- 2) Find x & y **intercepts**: $f(x) = 0$ & $f(0)$
- 3) If needed, find **asymptotes**
- 4) **Symmetry** $\rightarrow f(-x) = \begin{cases} f(x) & | & -f(x) \\ \text{sym. about } y\text{-axis} & | & \text{origin} \end{cases}$
- 5) a. **Inc/Dec** $\rightarrow$ Find crit. pts: $f'(x) = 0$ / DNE
 f'
 $\longleftrightarrow x$
 f''

b. **Concave up/down** $\rightarrow$ Inf. pts: $f''(x) = 0$ / DNE
- 6) **Local Extrema** $\rightarrow$ Use (5a): $+$ to $-$, local **MAX**
 $-$ to $+$, local **MIN**
- 7) **Connect** points with smooth curve based on 4-6



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EXAMPLE

Sketch a graph of the function f with the following properties.

- ☐ Continuous & differentiable everywhere
- ☐ x -intercepts at $x = 0$ and $x = 2$
- ☐ $f'(x) > 0$ on $(-\infty, 1)$ and $(2, \infty)$
- ☐ $f'(x) < 0$ on $(1, 2)$
- ☐ $f''(x) > 0$ for $x < 0.5$ & $x > 1.5$
- ☐ $f''(x) < 0$ for $0.5 < x < 1.5$

