TOPIC: TRIGONOMETRIC FUNCTIONS ON THE UNIT CIRCLE

Introduction to the Unit Circle

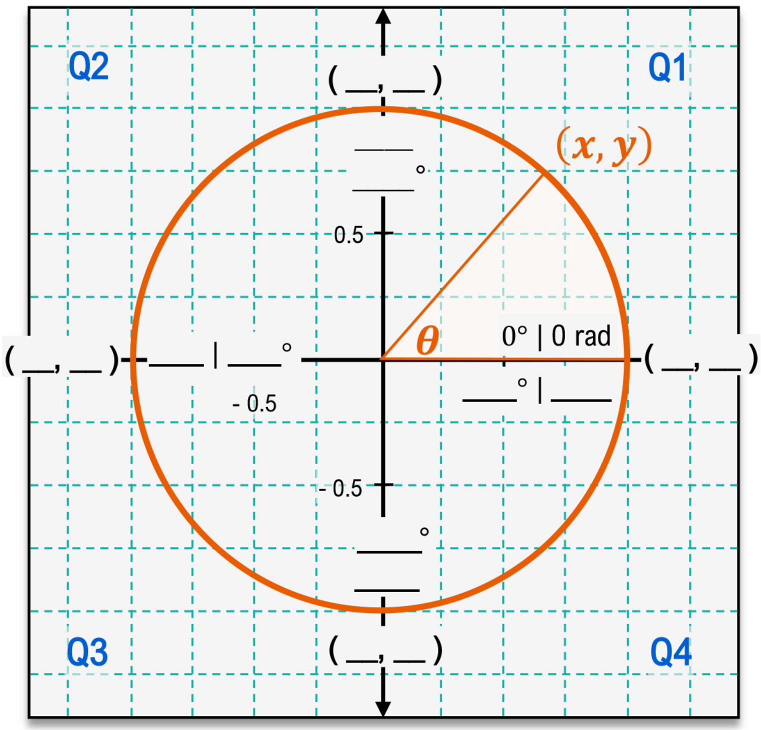
◆ **Unit Circle**: Circle of radius 1 relating angles from 0 to 360° (or ___ radians) to x & y values. Centered at (_____ ,_____).

Recall — **Circle**

$$(x-h)^2+(y-k)^2=r^2$$

New — **Unit Circle**

$$x^2+y^2=1$$



EXAMPLE Identify which points are on the unit circle and label them on the graph.

(A) $(1,1)$ **[ON | NOT ON]** unit circle

(B) $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ **[ON | NOT ON]** unit circle

PRACTICE Identify the quadrant that the given angle is located in.

(**A**) $\frac{7\pi}{4}$ radians

Quadrant: _____

(**B**) $\frac{\pi}{7}$ radians

Quadrant: _____

(**C**) $\frac{2\pi}{3}$ radians

Quadrant: _____

(**D**) $\frac{6\pi}{5}$ radians

Quadrant: _____

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PRACTICE Test whether the point is on the unit circle by plugging it into the equation.

$$\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$$

New **Unit Circle**

$$x^2 + y^2 = 1$$

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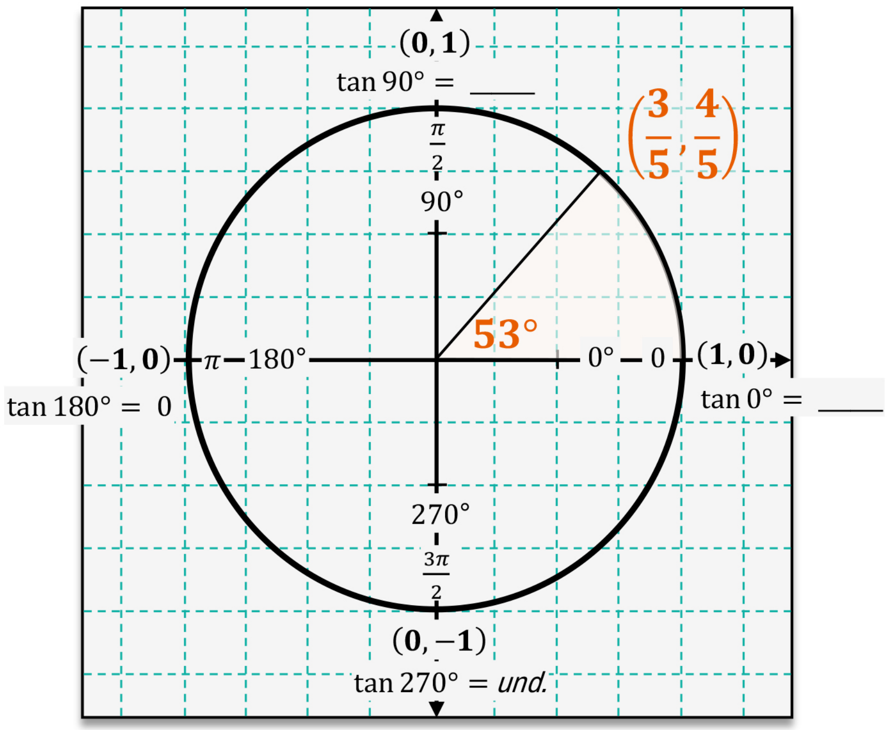
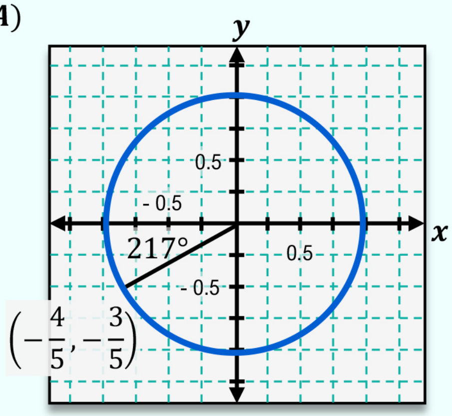
Sine, Cosine, & Tangent on the Unit Circle

- **Trigonometric Functions** relate angles to ________ on the unit circle. On the unit circle:
  - The **SIN** of an angle is *ALWAYS* the ___ value or the _________ of the corresponding triangle.
  - The **COS** of an angle is *ALWAYS* the ___ value or the _________ of the corresponding triangle.
  - The **TAN** of an angle is *ALWAYS* $\frac{}{}$



EXAMPLE Find the sine, cosine, and tangent of each angle using the unit circle.

(A)

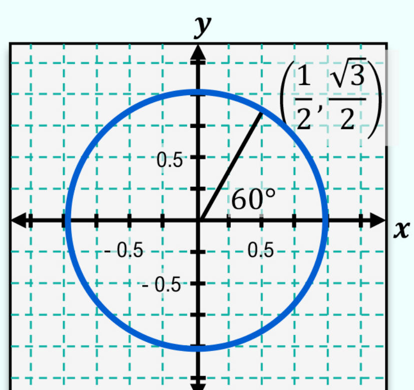


$\sin 217° =$ _____

$\cos 217° =$ _____

$\tan 217° =$ _____

(B)



$\sin 60° =$ _____

$\cos 60° =$ _____

$\tan 60° =$ _____

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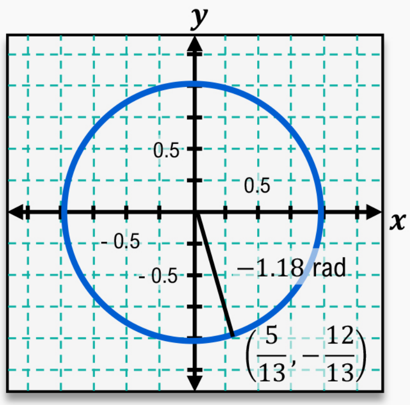
PRACTICE Find the sine, cosine, and tangent of each angle using the unit circle.

(***A***)

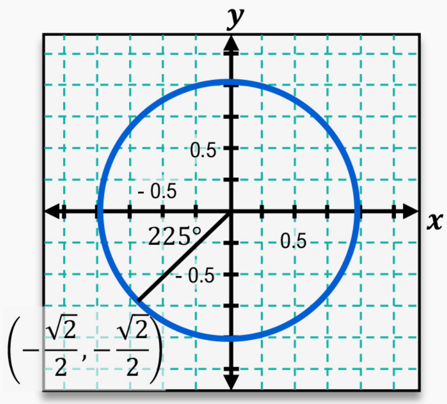


$\sin\theta =$ _____

$\cos\theta =$ _____

$\tan\theta =$ _____

(***B***)



$\sin\theta =$ _____

$\cos\theta =$ _____

$\tan\theta =$ _____

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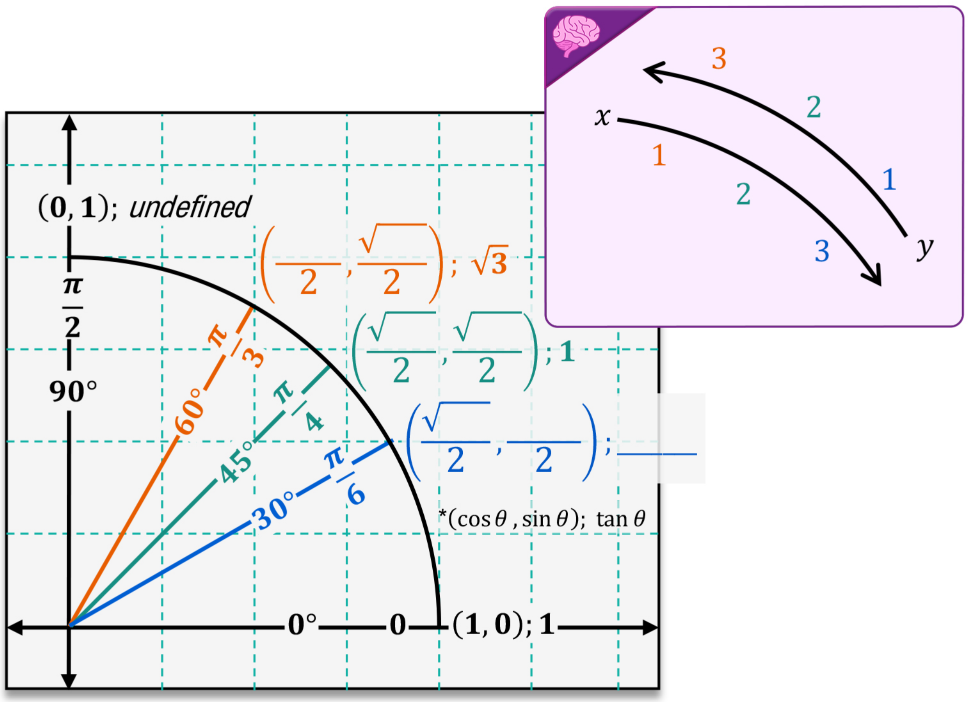
Sine, Cosine, & Tangent of 30°, 45°, & 60°

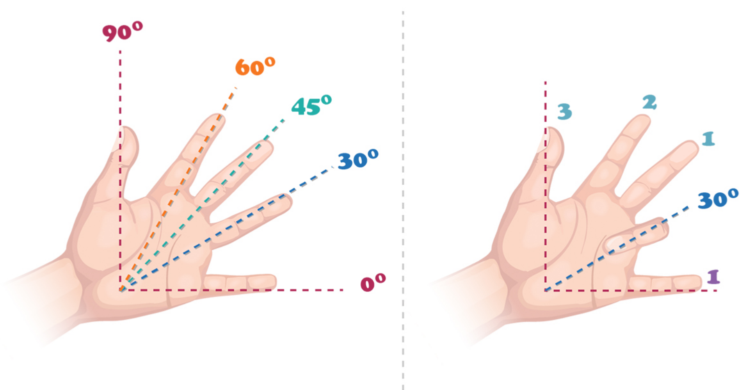
◆ You'll use trig values of 30°, 45°, & 60° (or $\frac{\pi}{6}$, $\frac{\pi}{4}$, & $\frac{\pi}{3}$) in most problems; Here's 2 methods to help memorize them.

- No matter how you choose to memorize these values, *ALWAYS* start with $\frac{\sqrt{}}{2}$

1) The 1-2-3 Rule



2) The Left Hand Rule



$$\cos\theta = \frac{\sqrt{\textit{fingers above}}}{2}$$

$$\sin\theta = \frac{\sqrt{\textit{fingers below}}}{2}$$

$$\tan\theta = \frac{\sqrt{\textit{fingers below}}}{\sqrt{\textit{fingers above}}}$$

$$\cos 30° = \frac{\sqrt{}}{2}$$

$$\sin 30° = \frac{\sqrt{}}{2} =$$

$$\tan 30° = \frac{\sqrt{}}{\sqrt{}} =$$

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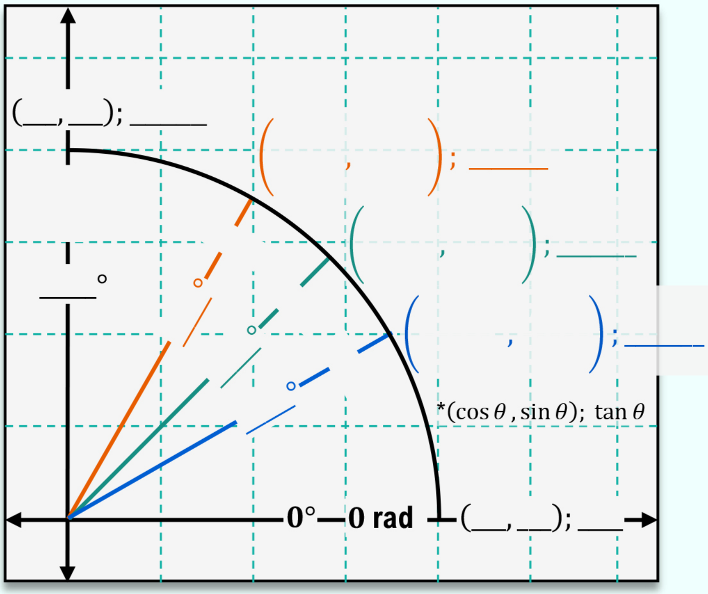
EXAMPLE Fill in all of the missing information.



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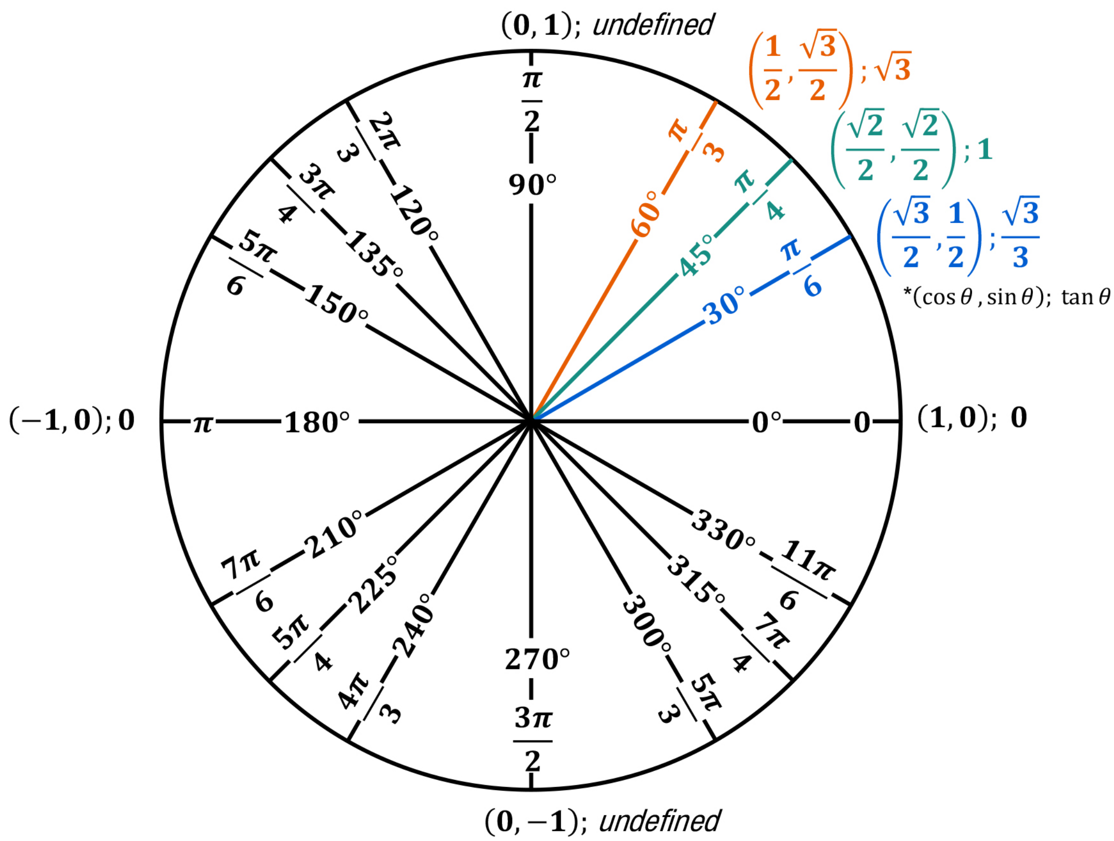
Reference Angles on the Unit Circle

◆ When given angles not in Q1, link them back to *known* Q1 angles (30°/45°/60°) by finding their **reference angle**.

- To do this, measure from the *given angle* directly to the ____________ part of the x-axis & write as a positive number.



PRACTICE Identify the reference angle of each given angle.

(***A***)	(***B***)	(***C***)
$120°$	$\frac{7\pi}{4}$	$210°$

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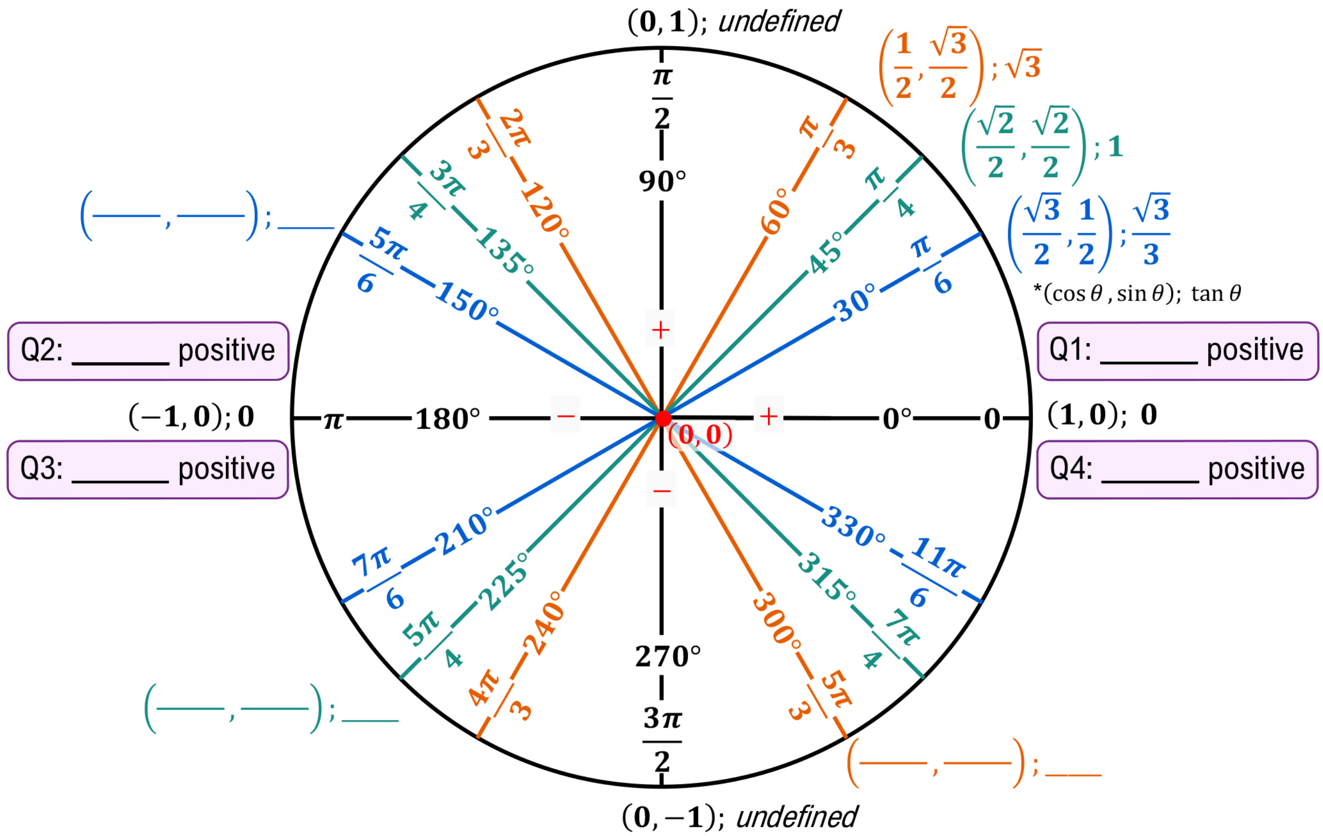
Trig Values in Quadrants II, III, & IV

- The $\sin$, $\cos$, & $\tan$ of angles *NOT* in Q1 have the same **value** as the $\sin$, $\cos$, & $\tan$ of their reference angles.
  - *HOWEVER*, the __________ of the values will change based on their quadrant.



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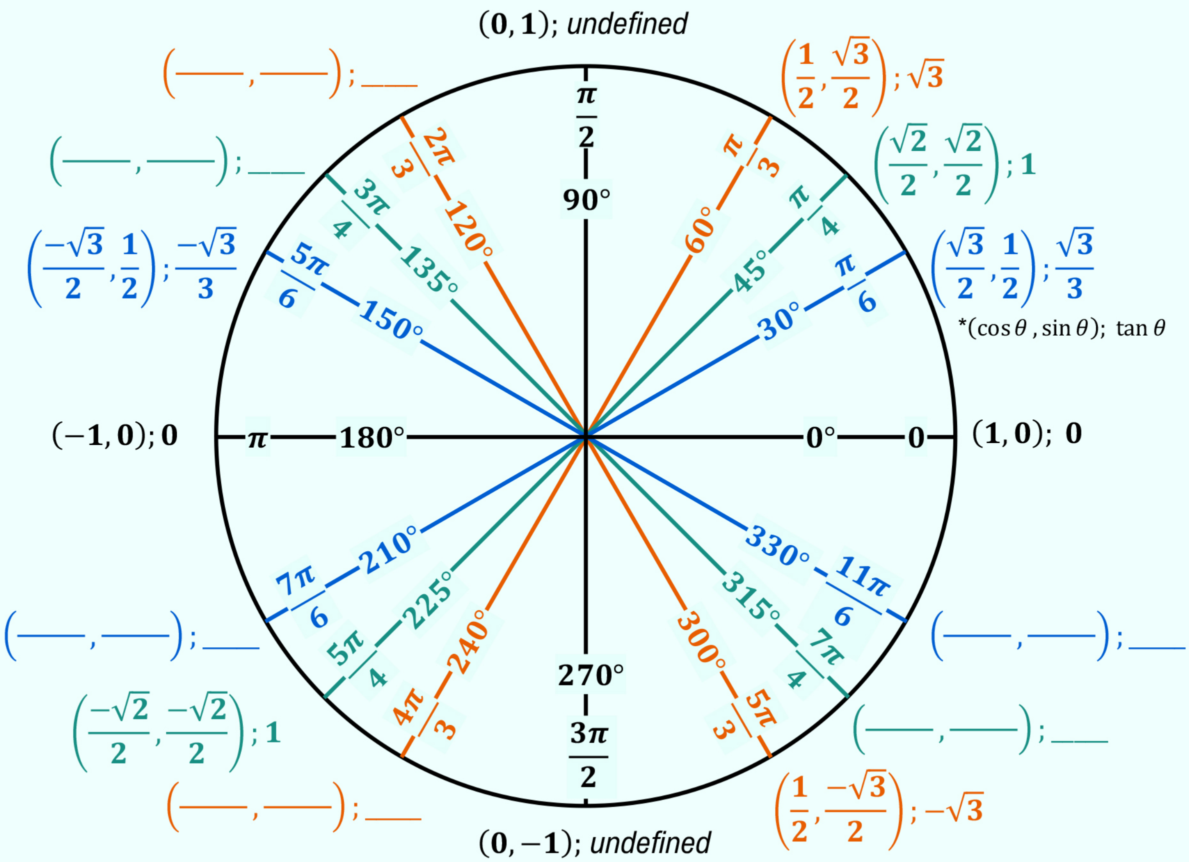
EXAMPLE Use reference angles to complete the missing trig values in quadrants II, III, & IV of the unit circle.



PRACTICE Identify what angle, θ, satisfies the following conditions.

(***A***)

$$\sin\theta=\frac{1}{2};\tan\theta<0$$

$\theta=$ ______

(***B***)

$$\cos\theta=\frac{\sqrt{3}}{2};\sin\theta<0$$

$\theta=$ ______

(***C***)

$$\tan\theta=-1;\cos\theta>0$$

$\theta=$ ______

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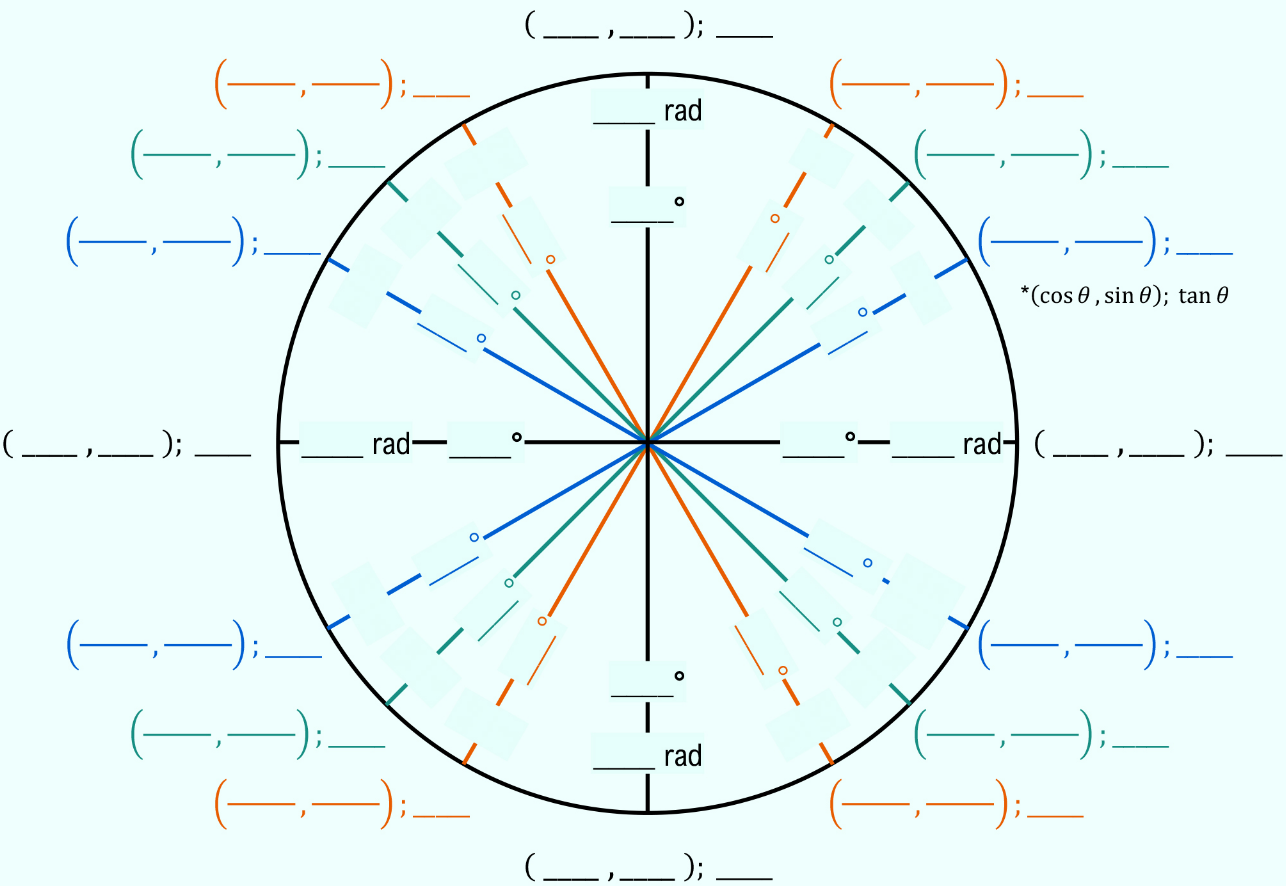
EXAMPLE Fill in all missing information in the unit circle below.



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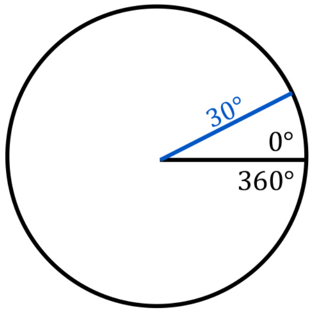
Coterminal Angles on the Unit Circle

- For angles > 360° or < 0°, use coterminal angles to find trig values, as they are equal to those on the unit circle.
  - **Coterminal Angle**: Angle with the same terminal side as another angle between 0 & 360°.
  - Find coterminal angles on the unit circle by adding/subtracting multiples of 360° (or 2π rad) to a given angle.



Recall Coterminal Angle

$$\theta_2 \pm 360° \cdot n = \theta_1$$

$390°$ __________ = ______

EXAMPLE Evaluate each trig function using coterminal angles on the unit circle.

(A)

$\tan 3\pi$

Coterminal Angle: ______

(B)

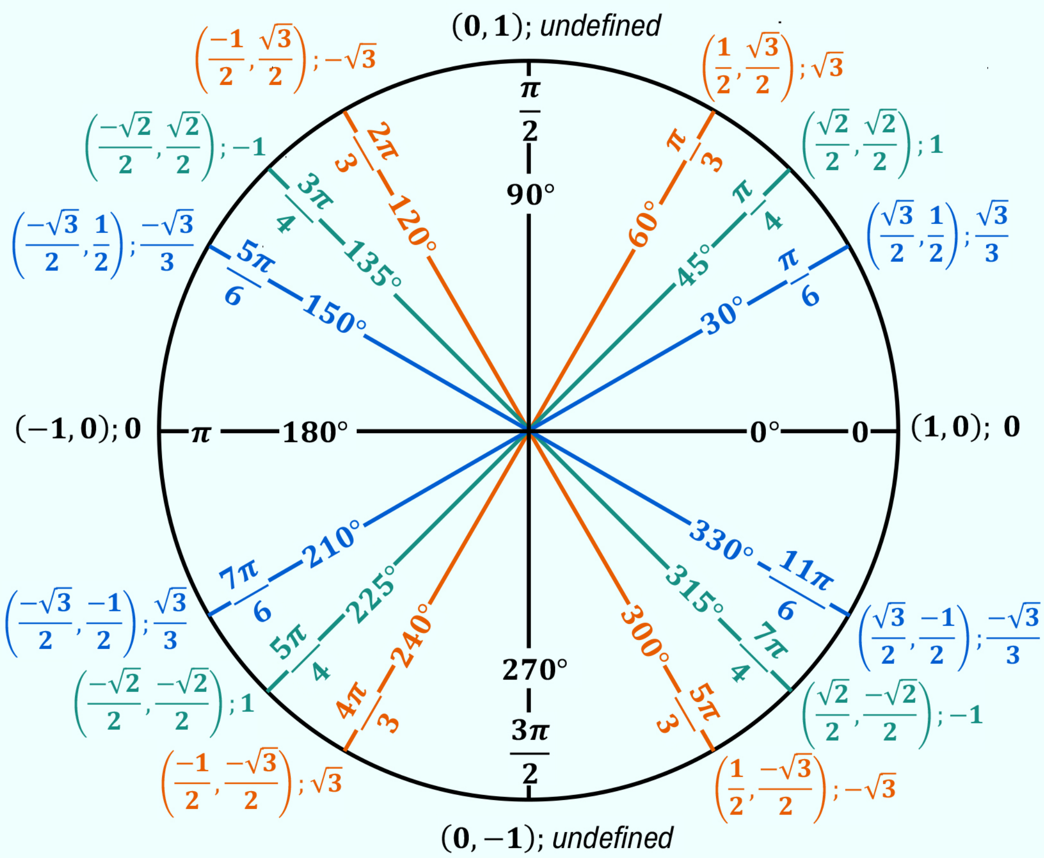
$\cos\left(-\frac{\pi}{4}\right)$

Coterminal Angle: ______

(C)

$\sin 390°$

Coterminal Angle: ______



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PRACTICE For each expression, identify which coterminal angle to use & determine the exact value of the expression.

(***A***)

$$\sin\frac{7\pi}{3}$$

Coterminal Angle: _____

(***B***)

$$\tan 765°$$

Coterminal Angle: _____

(***C***)

$$\cos\left(-\frac{10\pi}{4}\right)$$

Coterminal Angle: _____

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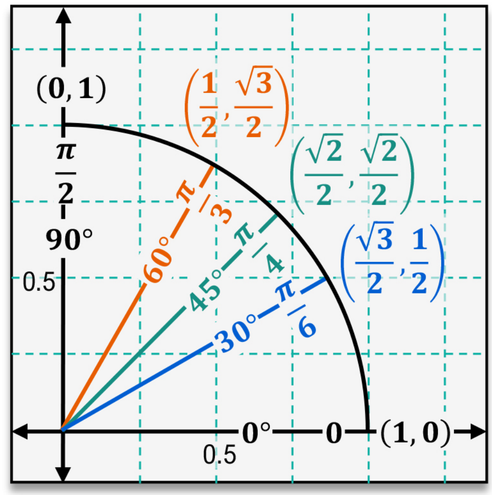
Cosecant, Secant, & Cotangent on the Unit Circle

- Recall: Besides Sine, Cosine, & Tangent, there are 3 other trig functions: Cosecant, Secant, & Cotangent.
  - These are the reciprocal trig functions, so find them by taking the reciprocal of sin, cos, & tan values on the unit circle.

Recall: SIN, COS, & TAN	New: SEC, CSC, & COT
$\sin\theta = \boldsymbol{y}$	$\csc\theta = \frac{1}{\sin\theta} = \frac{\mathbf{1}}{\_\_}$
$\cos\theta = \boldsymbol{x}$	$\sec\theta = \frac{1}{\cos\theta} = \frac{\mathbf{1}}{\_\_}$
$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\boldsymbol{y}}{\boldsymbol{x}}$	$\cot\theta = \frac{1}{\tan\theta} = \_\_$



EXAMPLE Evaluate each expression.

(**A**) $\csc\frac{\pi}{6}$

(**B**) $\cot\frac{\pi}{4}$

(**C**) $\sec 0$

PRACTICE Evaluate each expression.

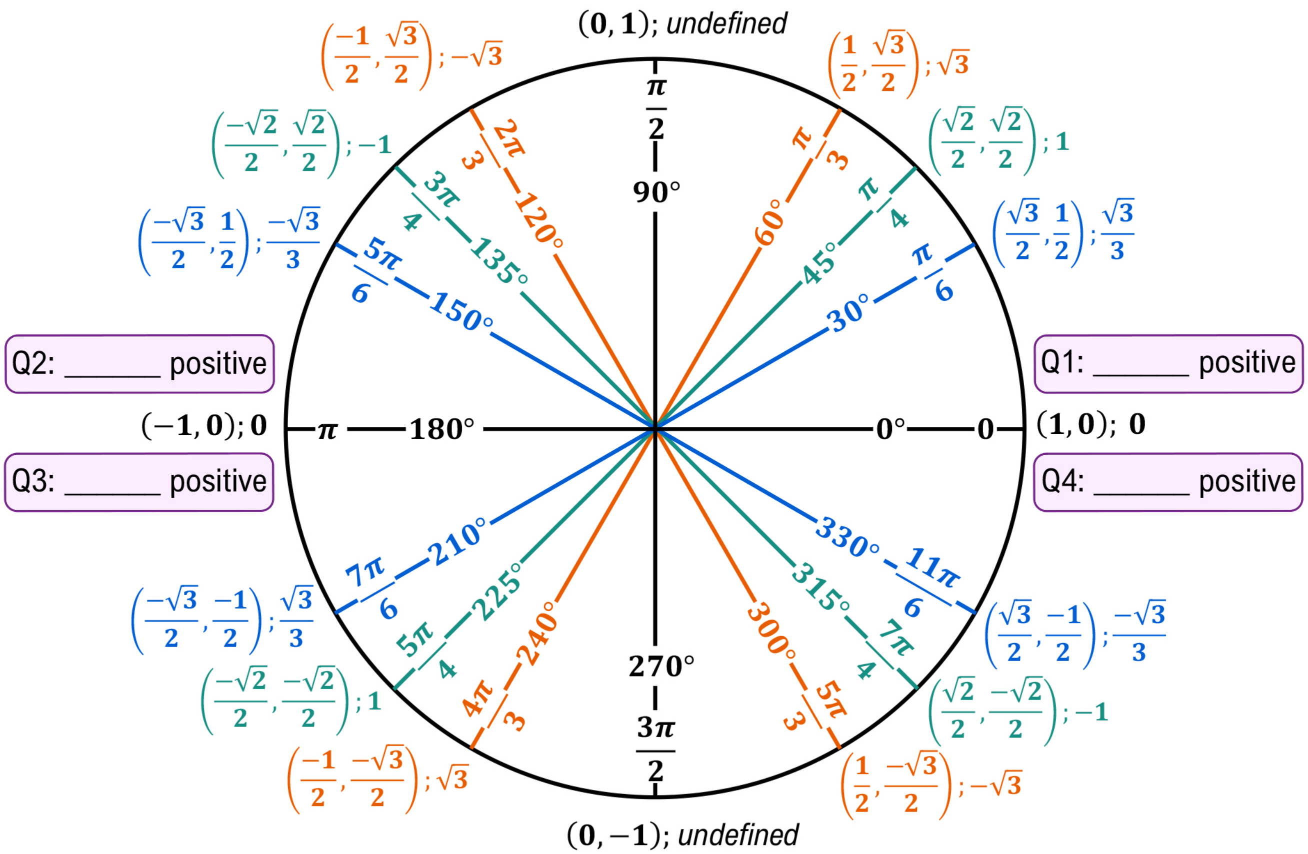
(**A**) $\cot\frac{11\pi}{6}$

(**B**) $\csc 225°$

(**C**) $\sec\frac{\pi}{3}$

TOPIC RESOURCE

The Unit Circle: Filled In



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The Unit Circle: Blank