TOPIC: INTRODUCTION TO LIMITS

Finding Limits Numerically and Graphically

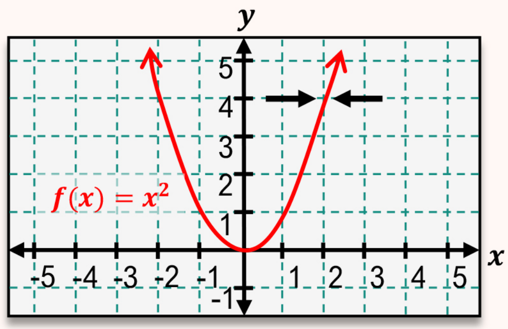
- A **limit** is the __________ a function $f(x)$ goes to as x gets *really close* to a given value.

New **Limits**

$\lim_{x \to c} f(x) = N$

"limit of $f(x)$ as x approaches c"

$\lim_{x \to 2} x^2 =$ _____



x	1.99	1.999	→ 2 ←	2.001	2.01
$f(x) = x^2$			[redacted]		

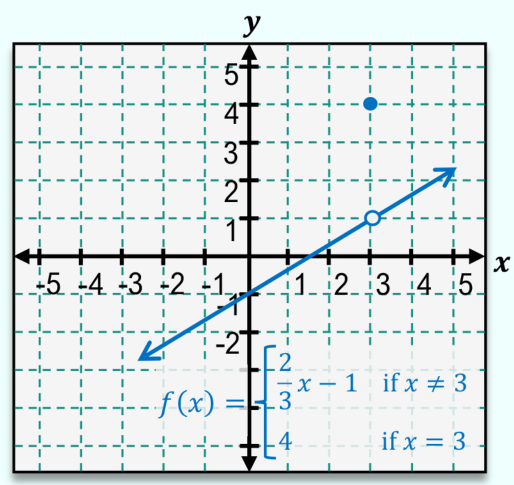
EXAMPLE Find each limit using the specified method.

(A) Find $\lim_{x \to 1}(x + 4)$ by creating a table of values.

x					
$f(x) = x + 4$					

$\lim_{x \to 1}(x + 4) =$ _____

(B) Find $\lim_{x \to 3} f(x)$ given the graph below.



$\lim_{x \to 3} f(x) =$ _____

- The limit will not always be equal to the function value ($\lim_{x \to c} f(x)$ may not be equal to $f(c)$).

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PRACTICE Find each limit by creating a table of values.

(A)

$$\lim_{x \to 0} -4x + 2$$

x					
$f(x)$					

(B)

$$\lim_{x \to 2} 3x^2 + 5x + 1$$

x					
$f(x)$					

(C)

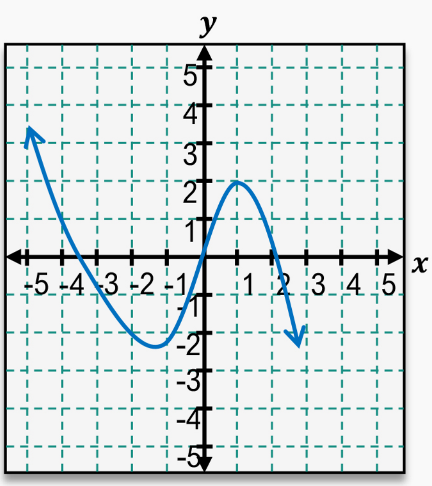
$$\lim_{x \to 1} \frac{x^2 - 4}{x - 2}$$

x					
$f(x)$					

PRACTICE Find each limit using the graph of $f(x)$ shown.

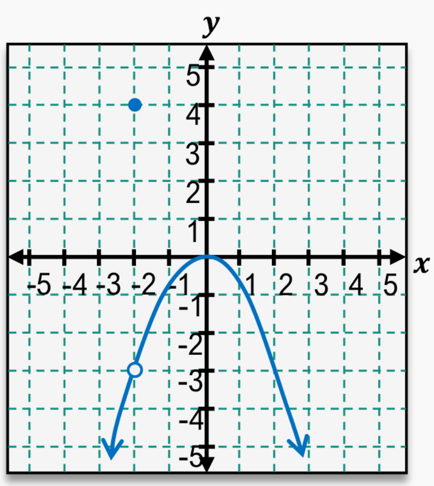
(A)

$$\lim_{x \to 1} f(x)$$



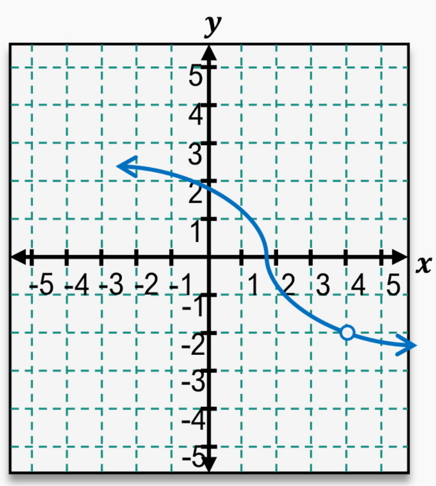
(B)

$$\lim_{x \to -2} f(x)$$



(C)

$$\lim_{x \to 4} f(x)$$



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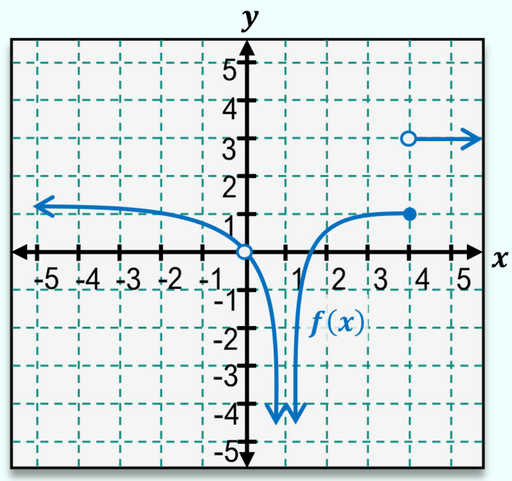
EXAMPLE Use the graph to estimate the value of each limit.

(A) $\lim_{x\to -2} f(x)$

(B) $\lim_{x\to 0} f(x)$

(C) $\lim_{x\to 3} f(x)$



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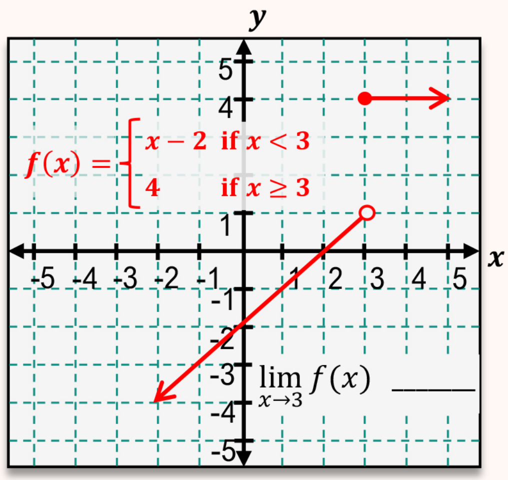
One-sided Limits

◆ The limit from each side is a **one-sided limit**, written as $\lim\limits_{x \to \_\_\_} f(x)$ (*from the left*) and $\lim\limits_{x \to \_\_\_} f(x)$ (*from the right*).

- If a function does *not* approach the SAME value from _______ sides the limit does *not* exist (DNE).

New **One Sided Limits**



$\lim\limits_{x \to 3^-} f(x) =$ ____ "from the left"

$\lim\limits_{x \to 3} f(x)$ ______

$\lim\limits_{x \to 3^+} f(x) =$ ____ "from the right"

x	2.99	2.999	→ 3 ←	3.001	3.01
$f(x)$					

EXAMPLE Use the graph to find the given limits.

(A)

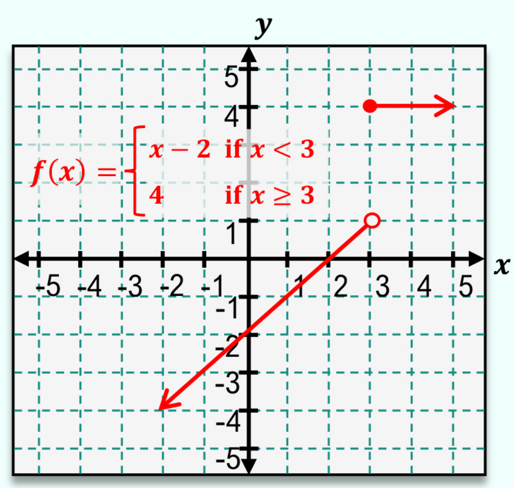
$\lim\limits_{x \to 1^-} f(x)$

(B)

$\lim\limits_{x \to 1^+} f(x)$

(C)

$\lim\limits_{x \to 1} f(x)$



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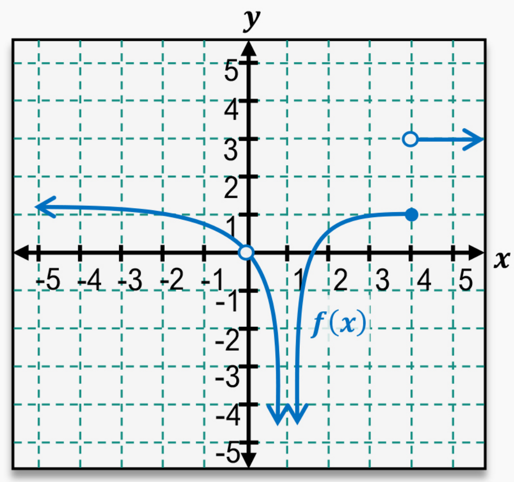
PRACTICE Using the graph, find the specified limit or state that the limit does not exist.

$(\boldsymbol{A})$

$\lim\limits_{x\to-2^-} f(x) =$ ____ $\lim\limits_{x\to-2^+} f(x) =$ ____ $\lim\limits_{x\to-2} f(x) =$ ____



$(\boldsymbol{B})$

$\lim\limits_{x\to0^-} f(x) =$ ____ $\lim\limits_{x\to0^+} f(x) =$ ____ $\lim\limits_{x\to0} f(x) =$ ____

$(\boldsymbol{C})$

$\lim\limits_{x\to4^-} f(x) =$ ____ $\lim\limits_{x\to4^+} f(x) =$ ____ $\lim\limits_{x\to4} f(x) =$ ____

PRACTICE Find the specified limit or state that the limit does not exist by creating a table of values.

$f(x) = \dfrac{1}{x}$

x						
$f(x)$						

$\lim\limits_{x\to1^-} f(x) =$ _____ $\lim\limits_{x\to1^+} f(x) =$ _____ $\lim\limits_{x\to1} f(x) =$ _____

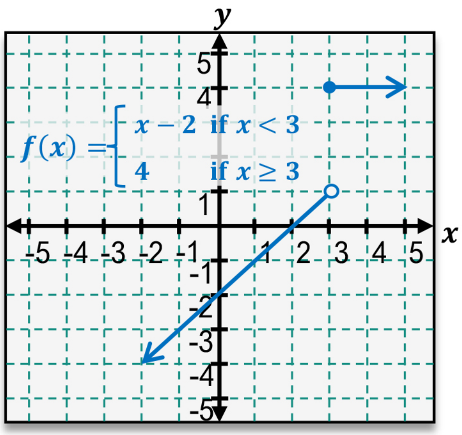
TOPIC: INTRODUCTION TO LIMITS

Cases Where Limits Do Not Exist

- Recall: If a function does not approach the same value from both sides the limit DNE.
  - The limit also DNE for a function that is ________________ as $x \to c$ or ________________ near c.
    (goes to _____ or ______)

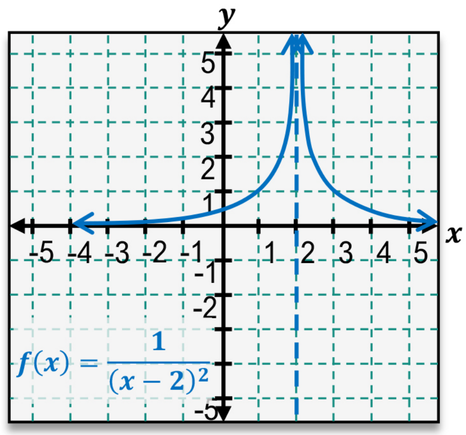
EXAMPLE Find the limits.

Piecewise with "Jump"



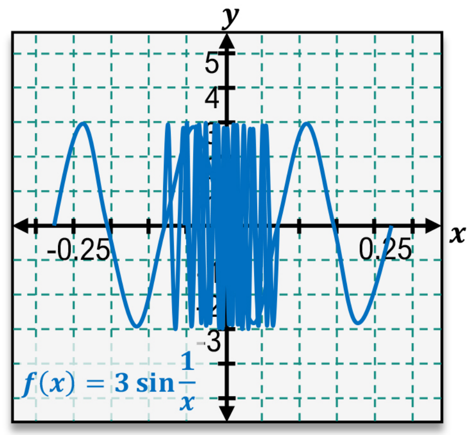
$\lim_{x \to 3} f(x)$ DNE

Unbounded



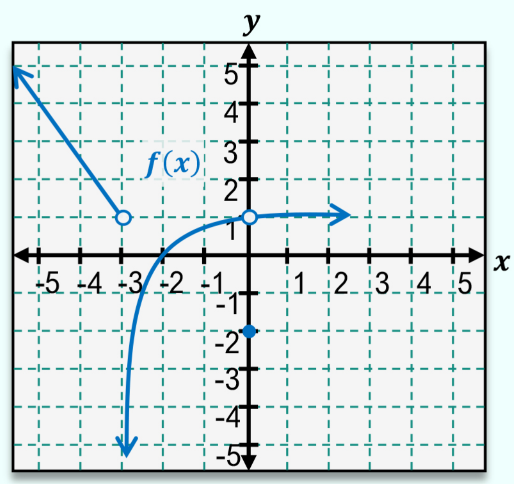
$\lim_{x \to 2} f(x)$ ________

Oscillating



$\lim_{x \to 0} f(x)$ ________

EXAMPLE Use the graph to find the given limits.



(A)

$\lim_{x \to -2} f(x)$

____

(B)

$\lim_{x \to 0} f(x)$

____

(C)

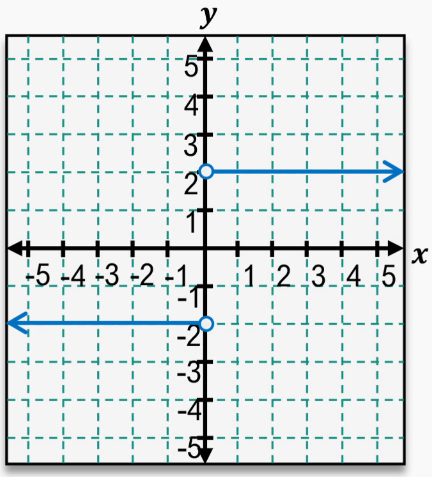
$\lim_{x \to -3} f(x)$

TOPIC: INTRODUCTION TO LIMITS

PRACTICE Use the graph for each $f(x)$ to estimate the value of each limit or state that it does not exist.

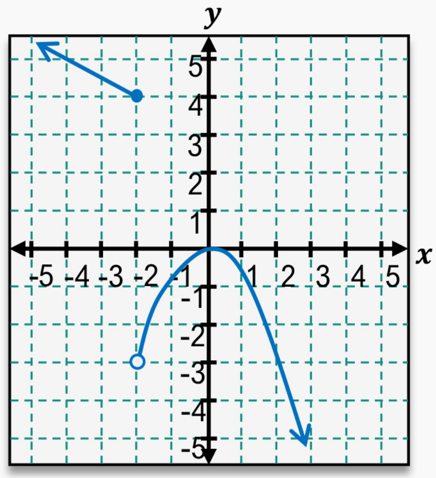
(A)

$$\lim_{x\to 1} f(x)$$



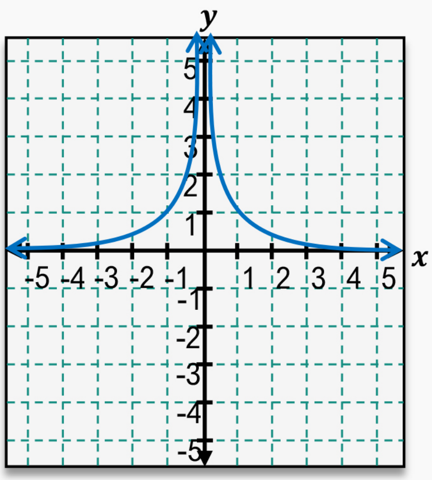
(B)

$$\lim_{x\to -2} f(x)$$



(C)

$$\lim_{x\to 0} f(x)$$



(D)

$$\lim_{x\to 0} f(x)$$