TOPIC: SUBSTITUTION FOR INDEFINITE INTEGRALS

Indefinite Integrals

Recall

$$\frac{d}{dx}\, f(g(x)) = f'(g(x)) \cdot g'(x)$$

◆ Recall: To evaluate *derivatives* of composite functions, we used the chain rule:

► To evaluate *integrals* with composite functions, use **substitution**.

New **Substitution**

$$\int f(g(x)) \cdot g'(x)\, dx = \int f(\_\_\_)\, \_\_\_$$

$$\int (x^2 + 1)^3 \cdot 2x\, dx = \int \_\_\_\_\_$$

$$= \_\_\_\_\_$$

$$= \_\_\_\_\_$$

EXAMPLE Evaluate the integral by making a substitution.

$$\int \sqrt{4x-1}\, dx = \int \sqrt{4x-1} \cdot \_\_\_\_\_\; dx$$

HOW TO: Evaluate Indefinite Integral with Substitution

1) Choose $u = g(x)$ (**inside** fcn), then find $du = g'(x)\, dx$

2) Rewrite int. ***only*** in terms of u & du; *If needed*: Mult. by _____ & _____

3) Integrate with respect to u

4) Replace u with $g(x)$

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EXAMPLE Use substitution to evaluate the given integral. Check your answer by differentiating.

$$\int (2x^3 + x + 7)^5 \, (6x^2 + 1) \, dx$$

HOW TO: Evaluate Indefinite Integral with Substitution

1) Choose $u = g(x)$ (**inside** fcn), then find $du = g'(x) \, dx$

2) Rewrite int. ***only*** in terms of u & du; *If needed*: Mult. by constant & reciprocal

3) Integrate with respect to u

4) Replace u with $g(x)$

EXAMPLE Evaluate the indefinite integral.

$$\int x^2 e^{x^3} \, dx$$

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PRACTICE Evaluate the indefinite integral.

(*A*)

$$\int 3t\sqrt{t^2+7}\,dt$$

HOW TO: Evaluate Indefinite Integral with Substitution

1) Choose $u = g(x)$ (**inside** fcn), then find $du = g'(x)\,dx$

2) Rewrite int. ***only*** in terms of u & du; *If needed*: Mult. by constant & reciprocal

3) Integrate with respect to u

4) Replace u with $g(x)$

(*B*)

$$\int \frac{1}{(3x+2)^5}\,dx$$

(*C*)

$$\int \frac{1}{2x+5}\,dx$$

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Substitution with an Extra Variable

◆ Recall: If du is missing a constant multiple, multiply by that constant & its reciprocal to make substitution work.

► If the integrand has an "extra x", rearrange $u = g(x)$ to get ___ in terms of ___ & replace in integral.

EXAMPLE Evaluate the integral by making a substitution.

$$\int x\sqrt{x+3}\ dx$$

HOW TO: Evaluate Indefinite Integral with Substitution

1) Choose $u = g(x)$ (**inside** fcn), then find $du = g'(x)\ dx$

2) Rewrite int. ***only*** in terms of u & du;
If needed: ► Mult. by constant & recip.
► Rewrite x in terms of u

3) Integrate with respect to u

4) Replace u with $g(x)$

TOPIC: SUBSTITUTION FOR INDEFINITE INTEGRALS

PRACTICE Evaluate the indefinite integral.

(A)

$$\int x(5+x)^{79}dx$$

HOW TO: Evaluate Indefinite Integral with Substitution

1) Choose $u = g(x)$ (**inside** fcn), then find $du = g'(x)\,dx$

2) Rewrite int. ***only*** in terms of u & du;
If needed: ▸ Mult. by constant & recip.
▸ Rewrite x in terms of u

3) Integrate with respect to u

4) Replace u with $g(x)$

(B)

$$\int \frac{t}{\sqrt{t-2}}dt$$

(C)

$$\int \frac{x}{(x-6)^5}dx$$

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EXAMPLE The marginal revenue of a coffee truck selling x lattes is given by the function below:

$$R'(x) = \frac{2x}{\sqrt{3x^2 - 50}}$$

$(\boldsymbol{A})$ Find the total revenue function $R(x)$ if the revenue from selling 15 lattes is \$75.

$(\boldsymbol{B})$ How many lattes must be sold for the truck to have a revenue of at least \$250?

EXAMPLE Answer the following questions based on the population growth rate given below, where $P(t)$ is the population of a particular species of fish present in a lake after t years.

$$P'(t) = 350 \cdot e^{t/3}$$

$(\boldsymbol{A})$ Find $P(t)$ given that there are 5000 fish in the lake at $t = 0$.

$(\boldsymbol{B})$ Find the population of fish after 4 years.