TOPIC: SUBSTITUTION

Definite Integrals

◆ To find *definite* integrals using substitution, there are two methods you can use:

► Method 1: Use substitution to solve as indefinite integral, then evaluate at *original* bounds.

EXAMPLE Evaluate the integral by making a substitution.

(A)

$$\int_0^2 (x^2+1)^3 \cdot 2x\,dx$$

$$\int \underbrace{(x^2+1)}_{u}{}^3 \cdot \underbrace{2x\,dx}_{du} = \int u^3\,du$$

HOW TO: Evaluate Definite Integrals with Substitution – Method 1

1) Choose $u = g(x)$ (**inside** fcn), then find $du = g'(x)\,dx$

2) Rewrite int. ***only*** in terms of u & du;
If needed: ► Mult. by constant & recip.
► Rewrite x in terms of u

3) Integrate with respect to u

4) Replace u with $g(x)$

5) Evaluate antiderivative at **original** bounds

► Method 2: Rewrite integrand in terms of u & du, solve definite integral, evaluating at *new* bounds $g(a)$ & $g(b)$.

New

$$\int_{\_\_}^{\_\_} f(g(x)) \cdot g'(x)\,dx = \int_{\_\_}^{\_\_} f(u)\,du$$

(B)

$$\int_0^2 \underbrace{(x^2+1)}_{u}{}^3 \cdot \underbrace{2x\,dx}_{du} = \int_{\_\_}^{\_\_} u^3\,du$$

HOW TO: Evaluate Definite Integrals with Substitution – Method 2

1) Choose $u = g(x)$ (**inside** fcn), then find $du = g'(x)\,dx$

2) a. Rewrite int. ***only*** in terms of u & du;
If needed: ► Mult. by constant & recip.
► Rewrite x in terms of u

b. Transform bounds: plug into $u = g(x)$

3) Integrate with respect to u

~~**4)** Replace u with $g(x)$~~

4) Evaluate antiderivative at **new** bounds

TOPIC: SUBSTITUTION

PRACTICE Evaluate the definite integral.

(*A*)

$$\int_0^1 \frac{t}{\sqrt{t^2+1}}\,dt$$

(*B*)

$$\int_1^2 (x-3)(x^2-6x)^7\,dx$$

(*C*)

$$\int_0^{\ln 2} \frac{e^{5y}}{3+e^{5y}}\,dy$$