

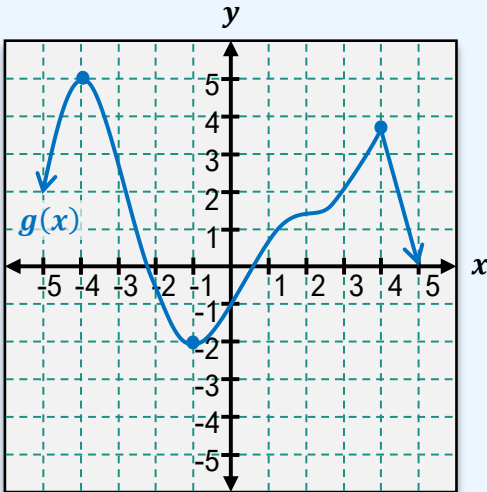
TOPIC: THE FIRST DERIVATIVE TEST

Critical Points

- ◆ Graphically, all local (relative) extrema either have a _____ tangent OR _____ tangent.
 - ▶ This means local extrema occur at **critical points**: where $f'(x) = \underline{\hspace{1cm}}$ OR $f'(x) \underline{\hspace{1cm}}$.

EXAMPLE

Find all the critical points of $g(x)$ & $f(x)$.

Recall	Local Extrema	New	Critical Points
			$f(x) = x^3 - 12x + 5$ $f'(x) = 0:$ $f'(x)$ DNE (for x in domain): Note: Not all critical points are local extrema.

- ◆ Critical points will be used to find many different characteristics of a function, including local & global extrema.

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EXAMPLE

Find the critical points of the given function $f(x)$.

$$f(x) = \frac{x^2}{x-4}$$

PRACTICE

Find the critical points of the given function.

(A)

$$g(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 12x$$

(B)

$$f(t) = \frac{6t}{t^2 + 1}$$

(C)

$$f(x) = \sqrt{2 - x^2}$$

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EXAMPLE

Find the critical points of the given function.

$$y = \ln(x - 1)$$

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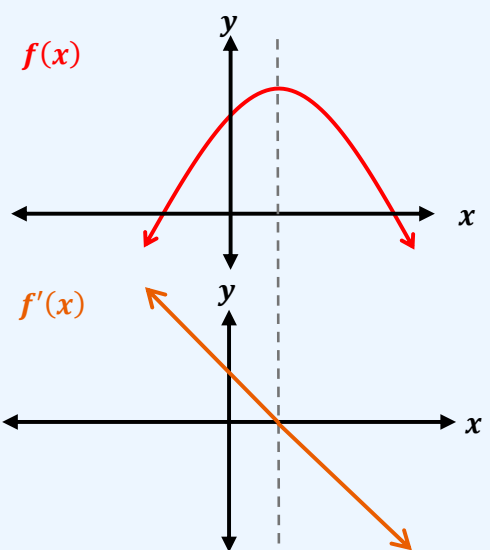
Determining Where a Function Is Increasing & Decreasing

◆ Recall: Looking at a graph from left to right, a function is increasing if it goes up & decreasing if it goes down.

► Given a function, find where it is **increasing** or **decreasing** based on the _____ of the _____.

EXAMPLE

Using the derivative, determine if $f(x)$ is increasing or decreasing at $x = 0$ & $x = 5$.

Recall	Increasing/Decreasing Graphically	New	Increasing/Decreasing Using Derivative
			$f(x) = -x^2 + 4x + 5$ $x = 0$: If f' is [+ -], f is [INC DEC] $x = 5$: If f' is [+ -], f is [INC DEC]

EXAMPLE

Determine the intervals for which $f(x)$ is increasing or decreasing.

$$f(x) = -x^2 + 4x + 5$$

←————→ x

HOW TO: Determine Intervals of Increase & Decrease

- Find critical points:
 $f'(x) = 0$ or $f'(x)$ DNE
- Make sign chart _____
based on critical points
- Plug value from each int. into f' :
If +, f _____ on interval
If -, f _____ on interval

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PRACTICE

Identify the open intervals on which the function is increasing or decreasing.

(A) $f(x) = 3x^4 + 8x^3 - 18x^2 + 7$

HOW TO: Determine Intervals of Increase & Decrease

- 1) Find critical points:
 $f'(x) = 0$ or $f'(x)$ DNE
- 2) Make sign chart **intervals**
based on critical points
- 3) Plug value from each int. into f' :
If +, f **INC** on interval
If -, f **DEC** on interval

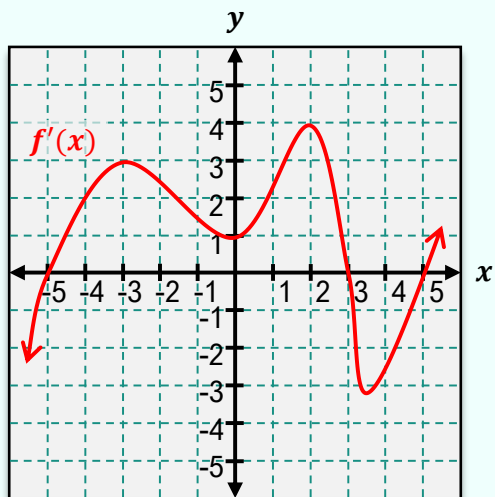
(B) $f(x) = x^{2/3} (4 - x)$

(C) $f(x) = xe^{-2x}$

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EXAMPLE

Identify the open intervals where $f(x)$ is increasing or decreasing based on the graph of $f'(x)$.



TOPIC: THE FIRST DERIVATIVE TEST

The First Derivative Test: Finding Local Extrema

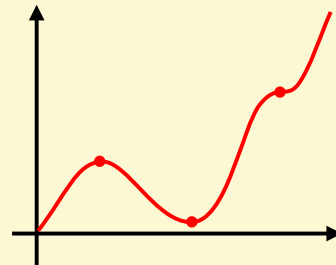
◆ Recall: The sign of the derivative tells us whether a function is increasing (f' is $+$) or decreasing (f' is $-$).

► Local extrema occur where the sign of the derivative f' _____.

First Derivative Test

Suppose c is a critical point of a continuous function f . If at c ,

- f' changes from $___ \rightarrow ___$, f has a local [**MAX** | **MIN**] at c .
- f' changes from $___ \rightarrow ___$, f has a local [**MAX** | **MIN**] at c .
- f' does $______$ change sign, f has $______$ local extrema at c .



EXAMPLE

Locate the local extrema of the function $f(x) = x^3 - 3x^2 + 4$.

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ &= 3x(x - 2) \end{aligned}$$

←————→ x

HOW TO: Find Local Extrema Using First Derivative Test

- 1) Find critical points:
 $f'(x) = 0$ or $f'(x)$ DNE
- 2) Make sign chart intervals based on critical points
- 3) Plug value from each int. into f'
If f' changes from:
 $+$ $\rightarrow$ $-$, crit. pt. is local **MAX**
 $-$ $\rightarrow$ $+$, crit. pt. is local **MIN**
- 4) If asked: Find **value** of max/min by plugging crit. pt. into $______$

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EXAMPLE

Identify the local minimum and maximum values of the given function, if any.

(A) $f(x) = (x + 7)^3$

HOW TO: Find Local Extrema Using First Derivative Test

- 1) Find critical points:
 $f'(x) = 0$ or $f'(x)$ DNE
- 2) Make sign chart intervals based on critical points
- 3) Plug value from each int. into f'
If f' changes from:
 $+$ $\rightarrow$ $-$, crit. pt. is local **MAX**
 $-$ $\rightarrow$ $+$, crit. pt. is local **MIN**
- 4) If asked: Find **value** of max/min by plugging crit. pt. into $f(x)$

(B) $h(t) = \frac{t^3}{2t + 1}$

TOPIC: THE FIRST DERIVATIVE TEST

PRACTICE

Identify the local minimum and maximum values of the given function, if any.

$$f(t) = t^2 \ln t, t > 0$$

HOW TO: Find Local Extrema Using First Derivative Test

- 1) Find critical points:
 $f'(x) = 0$ or $f'(x)$ DNE
- 2) Make sign chart intervals based on critical points
- 3) Plug value from each int. into f'
If f' changes from:
 $+$ $\rightarrow$ $-$, crit. pt. is local **MAX**
 $-$ $\rightarrow$ $+$, crit. pt. is local **MIN**
- 4) If asked: Find **value** of max/min by plugging crit. pt. into $f(x)$

EXAMPLE

Identify the global and local minimum and maximum values for the given function, if any.

$$f(x) = x^2 + 4x + 17$$