

TOPIC: IMPROPER INTEGRALS

Improper Integrals: Infinite Intervals

◆ An integral with _____ as one (or both) bounds is a type of *improper* integral.

New

Improper Integral with Infinite Bounds

If $f(x)$ is
continuous on... then...

(1) $[a, \infty)$

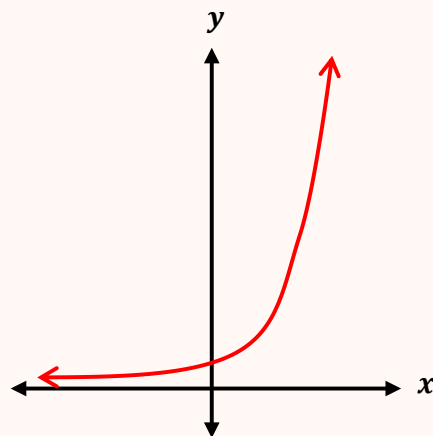
$$\int_a^{+\infty} f(x) dx = \lim_{t \rightarrow +\infty} \int_a^t f(x) dx$$

(2) $(-\infty, b]$

$$\int_{-\infty}^b f(x) dx = \lim_{t \rightarrow -\infty} \int_t^b f(x) dx$$

(3) $(-\infty, \infty)$

$$\int_{-\infty}^{+\infty} f(x) dx = \lim_{t \rightarrow -\infty} \int_t^c f(x) dx + \lim_{t \rightarrow +\infty} \int_c^t f(x) dx$$



◆ An integral is *convergent* if the limit _____ (a finite number) and *divergent* if the limit _____.

EXAMPLE

Determine whether each improper integral is convergent or divergent.

(A) $\int_0^{\infty} e^x dx$

[CONVERGENT | DIVERGENT]

(B) $\int_{-\infty}^0 e^x dx$

[CONVERGENT | DIVERGENT]

(C) $\int_{-\infty}^{\infty} e^x dx$

[CONVERGENT | DIVERGENT]

New

$$\int_a^{+\infty} f(x) dx = \lim_{t \rightarrow +\infty} \int_a^t f(x) dx$$

$$\int_{-\infty}^b f(x) dx = \lim_{t \rightarrow -\infty} \int_t^b f(x) dx$$

$$\int_{-\infty}^{+\infty} f(x) dx = \int_{-\infty}^c f(x) dx + \int_c^{\infty} f(x) dx$$

TOPIC: IMPROPER INTEGRALS

EXAMPLE

Determine whether the improper integrals diverge or converge. Evaluate the integral if it converges.

(A) $\int_0^{\infty} x e^x dx$

(B) $\int_{-\infty}^0 x e^x dx$

(C) $\int_{-\infty}^{\infty} x e^x dx$

TOPIC: IMPROPER INTEGRALS

PRACTICE

Evaluate the integral or state that it diverges.

(A)

$$\int_{-\infty}^{-1} \frac{2}{x^3} dx$$

(B)

$$\int_2^{\infty} \frac{1}{x(\ln x)^4} dx$$