TOPIC: SEQUENCES
Introduction to Sequences

$$\{2, 4, 6, 8, \_\_\_\}$$

- A **Sequence** is a LIST of numbers in a specific ____________.
  - The _______________ in a sequence are called **Terms** (a.k.a. "elements" or "members").
  - Sequences can be ***finite*** (________ after a certain number) or ***infinite*** (go on __________).

EXAMPLE Find the 5th term in each sequence & identify if the sequence is *finite* or *infinite*.

(A)

$$\{3, 6, 9, 12, \_\_\_, 18\}$$

[FINITE | INFINITE]

(B)

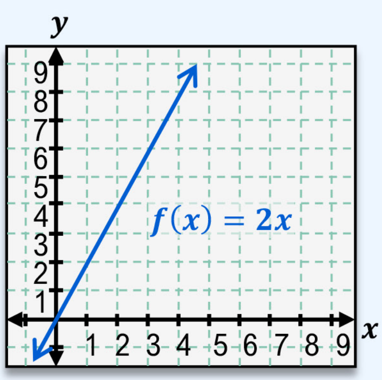
$$\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{}{}, \ldots\}$$

[FINITE | INFINITE]

- Sequences are like *functions*; they can follow _______ or equations.
  - Inputs are called **Indexes**, represented by ____, which *ALWAYS* starts at 1 and increases by 1
  - Outputs are called **Terms**, represented by ____.

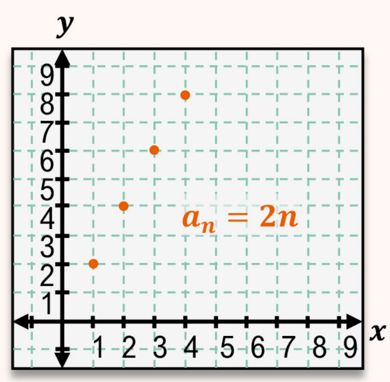
Recall — Functions

Inputs	x	-1	2	2.5	$\sqrt{7}$	π
Outputs	$f(x) = 2x$	-2	4	5	5.29	6.28



New — Sequences

Indexes	n	1	2	3	4	5
Terms	$a_n = 2n$					



EXAMPLE Find the first 3 terms in each sequence.

(A)

$$a_n = n^2$$

$a_1 =$ ____ , $a_2 =$ ____ , $a_3 =$ ____

(B)

$$a_n = \frac{1}{n+3}$$

$a_1 =$ ____ , $a_2 =$ ____ , $a_3 =$ ____

(C)

$$a_n = (-1)^n$$

$a_1 =$ ____ , $a_2 =$ ____ , $a_3 =$ ____

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PRACTICE The first 4 terms of a sequence are $\{\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}, \dots\}$. Continuing this pattern, find the 7th term.

PRACTICE Determine the first 3 terms of the sequence given by the general formula.

$$a_n = \frac{1}{n! + 1}$$

TOPIC: SEQUENCES
Writing a General Formula

- The **General** ("explicit") **Formula** of a sequence is an equation for a_n ("general term") containing n. $(n = 1, 2, 3, \ldots)$
  - To determine the general formula, find the _____________ between the numbers.

Common Patterns in General Formulas of Sequences

If sequences…	Increase by 1 or 2 or 3…	Alternate signs	Contain fractions	Increase exponentially
Formula contains…*	n or $2n$ or $3n\ldots$	$(-,+,-,\ldots) \rightarrow (-1)^n$ $(+,-,+,\ldots) \rightarrow (-1)^{n+1}$	Fractions (top & bottom may be different)	$(\#)^n$
EXAMPLE	$\{5, 6, 7, 8, 9\}$	$\{-5, 5, -5, 5, -5\}$	$\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots\right\}$	$\{2, 4, 8, 16, 32\}$

*Note: You will often have to adjust your formula by $+, -, \times, \div$ constants to get the desired sequence.

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EXAMPLE

Given the first 4 terms of a sequence shown below, write the general formula for the n^{th} term and use it to calculate the 15th term.

$$\{\frac{1}{1\cdot 2}, \frac{1}{2\cdot 3}, \frac{1}{3\cdot 4}, \frac{1}{4\cdot 5}, \ldots\}$$

EXAMPLE

Given the first 4 terms of a sequence shown below, write the general formula for the n^{th} term and use it to calculate the 18th term.

$$\{-2, 4, -6, 8, -10, \ldots\}$$

TOPIC: SEQUENCES
Recursive Formula

◆ Like general formulas, recursive formulas tell us how to find the n^{th} term in a sequence.

- *However*, **Recursive Formulas** show how to find $\boldsymbol{a_n}$ based on the _______________ term ($\boldsymbol{a_{n-1}}$) instead of $\boldsymbol{n}$.

Recall — **General Formula**

Indexes	n	1	2	3	4	5
Terms	$a_n = 2n$	$a_1 = 2$	$a_2 = 4$	$a_3 = 6$	$a_4 = 8$	$a_5 = 10$

$$\boldsymbol{a_n = 2n}$$

Need **[$\boldsymbol{\underline{n}}$ | PREVIOUS TERM]** to calculate n^{th} term

New — **Recursive Formula**

Indexes	n	1	2	3	4	5
Terms	$a_n = a_{n-1} + 2$	$a_1 = 2$				

$$\boldsymbol{a_n = a_{n-1} + 2}$$

Need **[$\boldsymbol{\underline{n}}$ | PREVIOUS TERM]** to calculate n^{th} term

EXAMPLE Given the recursive formula and first term of each sequence below, find the next 3 terms.

(***A***)

$$a_n = 2a_{n-1} + 3$$

$a_1 = 1$, $a_2 =$ ____ , $a_3 =$ ____ , $a_4 =$ ____

(***B***)

$$a_n = n \cdot a_{n-1}$$

$a_1 = 1$, $a_2 =$ ____ , $a_3 =$ ____ , $a_4 =$ ____

PRACTICE Write the first 6 terms of the sequence given by the recursive formula $a_n = a_{n-2} + a_{n-1}$; $a_1 = 1; a_2 = 1$

TOPIC: SEQUENCES
Arithmetic Sequences – Recursive Formula

- **Arithmetic Sequence:** Type of sequence where the _____________ between terms is *always* the ______ number.
  - This **common difference (d)** can be used to find additional terms using a recursive formula.

$$\{2, 7, 12, 17, \ldots\}$$

$a_1 =$ ___ $\qquad d =$ ___

New

$$a_n = a_{n-1} + \_\_\_$$

EXAMPLE For each sequence below, find the common difference and write the first 4 terms.

(A) $a_n = a_{n-1} + 4; \quad a_1 = 3$

(B) $a_n = a_{n-1} - 6; \quad a_1 = 9$

- To write a recursive formula from a given arithmetic sequence, first find the common difference.

EXAMPLE Write a recursive formula for the sequence

$$\{2, 5, 8, 11, 14\}$$

HOW TO: Write a Recursive Formula for Arithmetic Sequences

1) Find d by subtracting *any* 2 consecutive terms.

2) Write the formula, including the 1st term:

$$a_n = a_{n-1} + d; \quad a_1 = \#$$

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PRACTICE Write a recursive formula for the arithmetic sequence

$\{8, 2, -4, -10, \dots\}$

Recall

$a_n = a_{n-1} + d$

(Arithmetic, Recursive)

TOPIC: SEQUENCES
Arithmetic Sequences – General Formula

- The **General Formula** of arithmetic sequences gives the n^{th} term based on the ____ term & common difference d.
  - Remember: These equations allow you to calculate ***any*** terms *without* having to calculate previous terms!

Recall — Recursive Formula	New — General Formula
$\{2, 7, 12, 17, \ldots\}$	$\{2, 7, 12, 17, \ldots\}$
$\boldsymbol{a_n = a_{n-1} + 5}; \quad a_1 = 2$	$a_n = \_\_ + \_\_(\quad)$
$a_2 = a_1 + 5 = (2) + 5 = 7$	$a_2 = \cdots$
$a_3 = a_2 + 5 = (7) + 5 = 12$	$a_3 = \cdots$
$a_4 = a_3 + 5 = (12) + 5 = 17$	$a_4 = \_\_ + \_\_(\quad)$
$a_n = a_{n-1} + d$	$a_n = a_1 + d(\_\_\_\_\_\_)$

EXAMPLE For the sequence below, write a formula for the general or n^th term and use it to find the 101^st term.

$\{2, 5, 8, 11, 14\}$

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PRACTICE Find the general formula for the arithmetic sequence below. Without using a recursive formula, calculate the 30th term.

$$\{-9, -4, 1, 6, \ldots\}$$

Recall

$$a_n = a_1 + d(n-1)$$

(Arithmetic, General)

EXAMPLE Write a general formula from the recursive formula below. What would be the 15th term in this sequence?

$$a_n = a_{n-1} + 3; \quad a_1 = 2$$

Recall

$$a_n = a_1 + d(n-1)$$

(Arithmetic, General)

TOPIC: SEQUENCES

EXAMPLE The 4th & 6th terms of a sequence are $a_4 = -2$ and $a_6 = 6$. Find the 18th term of the sequence.

Recall
$a_n = a_1 + d(n-1)$
(Arithmetic, General)

TOPIC: SEQUENCES
Geometric Sequences – Recursive Formula

- **Geometric Sequence**: Type of sequence where the ___________ between terms is *always* the _______ number.
  - This **common ratio (r)** can be used to find additional terms using a recursive formula.

Recall Arithmetic Sequence	**New** Geometric Sequence
$\{3, 6, 9, 12, \dots\}$	$\{3, 9, 27, 81, \dots\}$
$a_1 = 3$ $a_n = a_{n-1} + 3$	$a_1 = 3$ $a_n =$ ____________
$a_n = a_{n-1} + d$	$a_n = a_{n-1} \cdot$ __
[+ | ×] number to get next term Grow **[SLOWER | FASTER]**	**[+ | ×]** number to get next term Grow **[SLOWER | FASTER]**

- To write a recursive formula for a geometric sequence, first find the common ratio.

EXAMPLE Write a recursive formula for the sequence.

$$\{5, 20, 80, 320, \dots\}$$

HOW TO: Write a Recursive Formula for Geometric Sequences

1) Find r by dividing *any* 2 consecutive terms.

2) Write the formula, including the 1st term:

$$a_n = a_{n-1} \cdot r; \quad a_1 = \#$$

TOPIC: SEQUENCES

PRACTICE Write a recursive formula for the arithmetic sequence

$$\{18, 6, 2, \frac{2}{3}, \ldots\}$$

Recall

$a_n = a_{n-1} \cdot r$

(Geometric, Recursive)

TOPIC: SEQUENCES
Geometric Sequences – General Formula

◆ The **General Formula** of geometric sequences gives the n^{th} term based on the ____ term & common ratio r.

- Remember: These equations allow you to calculate ***ANY*** terms *without* having to calculate previous terms!

	Recall – Recursive Formula	New – General Formula
Arithmetic	$a_n = a_{n-1} + d$	$a_n = a_1 + d(n-1)$
Geometric	$\{3, 6, 12, 24, \ldots\}$ $a_n = a_{n-1} \cdot 2; \quad a_1 = 3$ $a_2 = a_1 \cdot 2 = (3) \cdot 2 = 6$ $a_3 = a_2 \cdot 2 = (6) \cdot 2 = 12$ $a_4 = a_3 \cdot 2 = (12) \cdot 2 = 24$ $a_n = a_{n-1} \cdot r$	$\{3, 6, 12, 24, \ldots\}$ $a_n = \_\_\_ \cdot \_\_\_^{(\_\_\_)}$ $a_2 = \cdots$ $a_3 = \cdots$ $a_4 = \_\_\_ \cdot \_\_\_^{(\_\_\_)}$ $a_n = a_1 \cdot r^{(\_\_\_\_)}$

EXAMPLE For each sequence below, write a formula for the general or n^{th} term and use it to find the 12^{th} term.

$\{5, 20, 80, 320, \ldots\}$

TOPIC: SEQUENCES

PRACTICE Find the 10th term of the geometric sequence in which $a_1 = 5$ and $r = 2$.

Recall
$a_n = a_1 \cdot r^{n-1}$

(Geometric, General)

PRACTICE Write a formula for the general or n^{th} term of the geometric sequence where $a_7 = 1458$ and $r = -3$.

Recall
$a_n = a_1 \cdot r^{n-1}$

(Geometric, General)