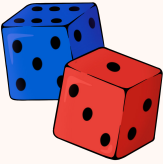





TOPIC: CONTINUOUS PROBABILITY MODELS

Continuous and Discrete Random Variables

◆ A *random variable* is a function that assigns a numerical value to the _____ of an experiment.

▶ Random variables can be either **discrete** or **continuous**.

Discrete Random Variables	Continuous Random Variables
<p style="text-align: center;">[COUNTABLE UNCOUNTABLE]</p> <p>Outcomes are _____</p> <p style="text-align: center;"><i>Examples:</i></p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p><i>Dice Roll</i></p> </div> <div style="text-align: center;">  <p><i># of students in class</i></p> </div> </div>	<p style="text-align: center;">[COUNTABLE UNCOUNTABLE]</p> <p>Outcomes are _____</p> <p style="text-align: center;"><i>Examples:</i></p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p><i>Temperature</i></p> </div> <div style="text-align: center;">  <p><i>Time</i></p> </div> </div>

EXAMPLE

Classify each random variable as either discrete or continuous.

(A) Number of questions scored correctly on a 10-question quiz.

[DISCRETE | CONTINUOUS]

(B) Exact amount of milk in a jug.

[DISCRETE | CONTINUOUS]

(C) Speed of a car on an interstate.

[DISCRETE | CONTINUOUS]

(D) Number of texts received in one hour.

[DISCRETE | CONTINUOUS]

TOPIC: CONTINUOUS PROBABILITY MODELS

PRACTICE

Which of the following is a discrete random variable?

- (A) The amount of milk in a jug (in gallons)
- (B) The number of times a die shows 4 in 10 rolls
- (C) The height (in cm) of a student
- (D) The distance a car travels before running out of fuel

PRACTICE

Which of the following is a continuous random variable?

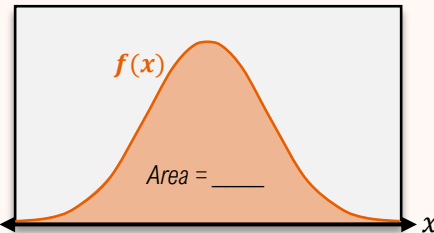
- (A) The number of goals scored in a soccer match
- (B) The number of cars passing through a toll booth in an hour
- (C) The number of students present in class
- (D) The amount (in inches) of rainfall in a month

TOPIC: CONTINUOUS PROBABILITY MODELS

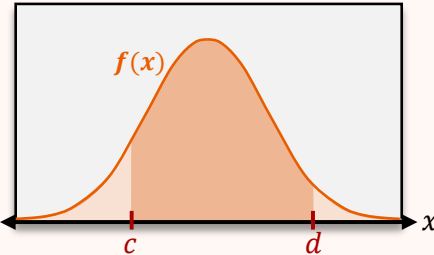
Probability Density Function (PDF)

- ◆ A PDF, $f(x)$, gives the likelihood of a _____ random variable X taking on a value within an interval.
 - ▶ The probability of X occurring is an _____ sum and requires integration over a certain interval.

NewProbability Density Function (PDF)



The graph shows a bell-shaped curve $f(x)$ on a coordinate system with the x-axis. The area under the curve is shaded orange and labeled "Area = _____".



The graph shows the same bell-shaped curve $f(x)$. Two vertical red lines are drawn at $x=c$ and $x=d$ on the x-axis. The area under the curve between these two lines is shaded orange.

$f(x)$ is a PDF if:

1. $f(x) \geq 0$ for all x
2. $\int_{-\infty}^{\infty} f(x) dx = 1$
3. The probability that X lies on the interval $[c, d]$ is:
$$P(c \leq X \leq d) = \int_c^d f(x) dx$$

EXAMPLE

Answer the following about the probability density function $f(x) = \begin{cases} \frac{3}{2}x^2 + 2x^3 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$.

(A) Verify that the probability function f satisfies the first two conditions.

1. $f(x) \geq 0$ for all x

2. $\int_{-\infty}^{\infty} f(x) dx = 1$

(B) Find $P(0.1 \leq X \leq 0.3)$.

TOPIC: CONTINUOUS PROBABILITY MODELS

EXAMPLE

Answer the following about the probability density function $f(x) = \begin{cases} \frac{3}{2}x^2 + 2x^3 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$.

(A) Find $P(X = .5)$

(B) Find $P(X \leq .6)$

TOPIC: CONTINUOUS PROBABILITY MODELS

EXAMPLE

A company manufactures rechargeable batteries. The lifetime (in years) of a certain battery is modeled by the probability density function $f(x) = \begin{cases} 2(1-x) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

(A) Verify that f is a probability density function.

(B) What is the probability that the battery lasts at least 0.75 years?

(C) What is the probability that the battery lasts between 0.2 and 0.6 years?

EXAMPLE

Find a value of k that will make $f(x) = kx^3$ a probability density function on the interval $[1,3]$.

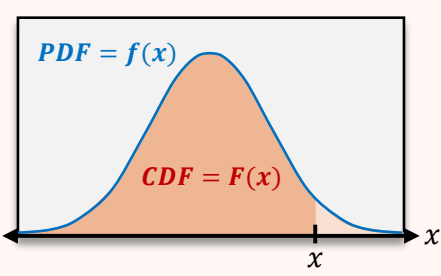
TOPIC: CONTINUOUS PROBABILITY MODELS

Cumulative Distribution Function (CDF)

- ◆ Recall: A **PDF**, $f(x)$, describes the likelihood of a continuous random variable X taking on a value within an interval.
 - ▶ A **CDF**, $F(x)$, gives the probability that X is *less than or equal to* a certain value.

New Cumulative Distribution Function (CDF)

If f is a **probability density function**, then the **cumulative distribution function** F is:


$$F(x) = P(X \leq x) = \int f(t) dt$$

Furthermore, $P(c \leq X \leq d) = \text{_____} - \text{_____}$

Properties of the CDF:

2. $F'(x) = f(x)$ wherever f is continuous.
3. $0 \leq F \leq 1, -\infty < x < \infty$
4. F never _____

EXAMPLE

Answer the following given the probability density function $f(x) = \begin{cases} 3x^2 e^{-x^3} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

(A) Find the cumulative distribution function.

(B) Find $P(1 \leq X \leq 3)$.

TOPIC: CONTINUOUS PROBABILITY MODELS

EXAMPLE

Suppose that the amount of time (in hours) that students need to finish a standardized test is modeled by the probability density function $f(t) = 2t$ for $0 \leq t < 1$.

(A) Find the cumulative distribution function.

(B) Use the CDF to determine the probability that a randomly chosen student finishes the test in 45 minutes or less.

TOPIC: CONTINUOUS PROBABILITY MODELS

EXAMPLE

The cumulative distribution function $F(x)$ is given below. Find the probability density function, $f(x)$, associated w/ $F(x)$.

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x^2}{4} & \text{if } 0 \leq x \leq 2 \\ 1 & \text{if } x > 2 \end{cases}$$

PRACTICE

Find the cumulative distribution function for the probability density function $f(x) = \frac{3}{8}(x - 1)^2$ on the interval $[1,3]$.
Use the CDF to find the probability that X is between 2 and 2.5.