TOPIC: SERIES

Intro to Series: Partial Sums

◆ The sum of the first n terms of a _______________, $\{a_n\}$ is called the n^{th} partial sum, s_n.

EXAMPLE Given the sequence $\{a_n\} = \frac{1}{2^n}$, find ($\boldsymbol{A}$) the first 5 terms of $\{a_n\}$ and ($\boldsymbol{B}$) the first 5 partial sums and the n^{th} partial sum.

Recall: Sequences	New: Partial Sums (Finite Series)
$\{a_n\} = \{a_1, a_2, a_3, \dots\}$	$s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$
$a_1 =$	$s_1 =$
$a_2 =$	$s_2 =$
$a_3 =$	$s_3 =$
$a_4 =$	$s_4 =$
$a_5 =$	$s_5 =$
	$s_n =$

◆ The partial sums $s_1, s_2, s_3, \dots s_n$ form a *new* sequence, $\{s_n\}$.

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EXAMPLE

Find a formula for the nth partial sum S_n of the given sequence $a_n = \frac{1}{n} - \frac{1}{n+1}$. Use this formula to find the sum of the first 15 terms.

PRACTICE

Compute the first four partial sums and find a formula for the nth partial sum.

$$\sum_{n=1}^{\infty} 2n - 1$$

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Convergence of an Infinite Series

◆ An infinite series is the sum of infinitely many terms in a sequence

New

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots = \lim_{n\to\infty} s_n$$

► If $\lim_{n\to\infty} s_n$ is finite, the series **[CONVERGES to** $\lim_{n\to\infty} s_n$ **| DIVERGES]**.

► If $\lim_{n\to\infty} s_n$ DNE, the series **[CONVERGES to** $\lim_{n\to\infty} s_n$ **| DIVERGES]**.

EXAMPLE Find the series' sum if it converges, or state that it diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n+1} - \frac{1}{n+2}$$

[CONVERGES to ________ | DIVERGES]

◆ If $\sum a_n$ and $\sum b_n$ are convergent series, then $\sum a_n \quad b_n$ and $\sum \quad \cdot a_n$ are also convergent.

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EXAMPLE Find a formula for the n th partial sum and use it to find the series' sum (if it converges).

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots\frac{1}{n\cdot(n+1)}+\cdots$$

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Geometric Series

◆ Recall: A geometric sequence has a common ratio r and is in the form

Recall
$a_n = a \cdot r^{n-1}$

CONVERGENCE TESTS			
Name	**Series**	**Converges if…**	**Diverges if…**
Geometric Series	$\sum_{n=1}^{\infty} a \cdot r^{n-1}$ or $\sum_{n=0}^{\infty} a \cdot r^{n}$	$\|r\|$ 1 & sum is: $S = \frac{a}{1-r}$	$\|r\|$ 1

EXAMPLE Determine whether the given series are convergent. If so, find the sum.

$(\boldsymbol{A})$

$$\sum_{n=0}^{\infty} 3\left(\frac{2}{5}\right)^n$$

$(\boldsymbol{B})$

$$1 - \left(\frac{3}{e}\right) + \left(\frac{3}{e}\right)^2 - \left(\frac{3}{e}\right)^3 + \cdots$$

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EXAMPLE Determine whether the given series are convergent. If so, find the sum.

$(\boldsymbol{A})$

$$\sum_{n=0}^{\infty} \frac{3}{4^n}$$

$(\boldsymbol{B})$

$$\sum_{n=0}^{\infty} \frac{1}{2^n} + \frac{5}{3^n}$$

$(\boldsymbol{C})$

$$\frac{1}{9} - \frac{1}{3} + 1 - 3 + 9 - \cdots$$