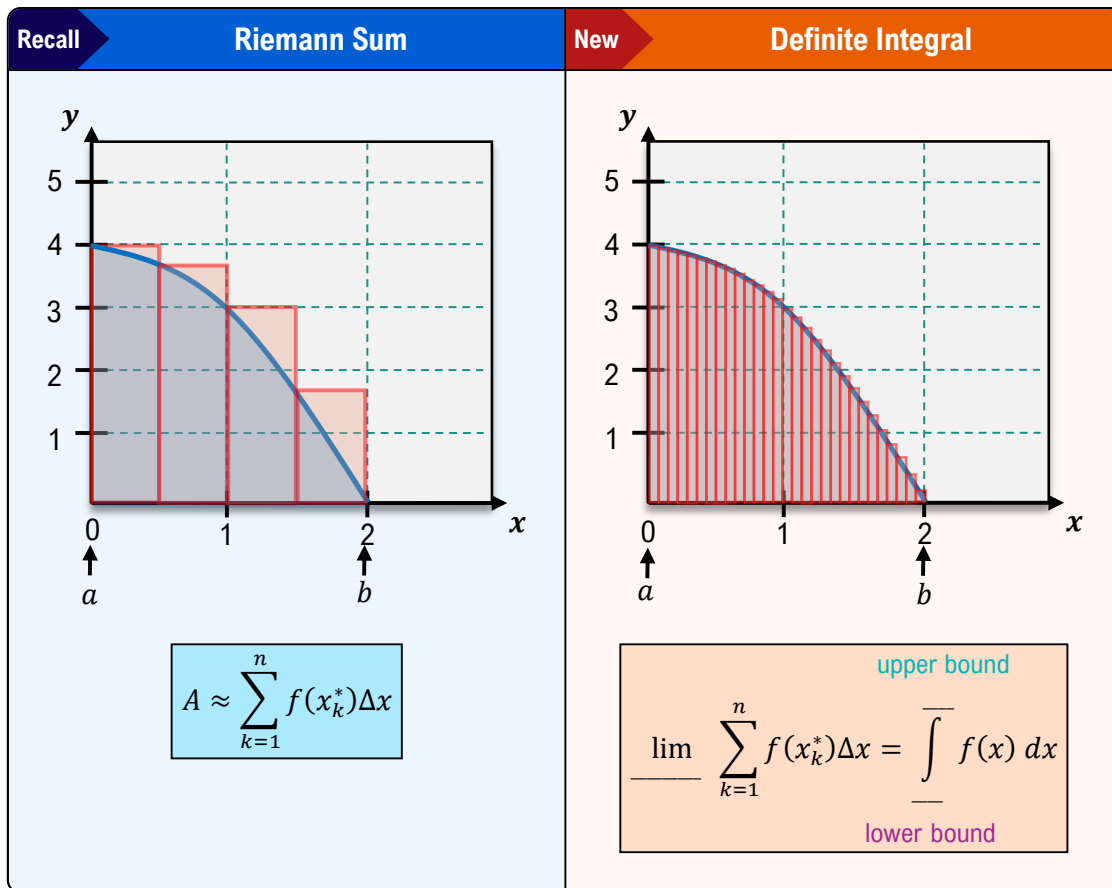


TOPIC: INTRODUCTION TO THE DEFINITE INTEGRAL

Definition of the Definite Integral

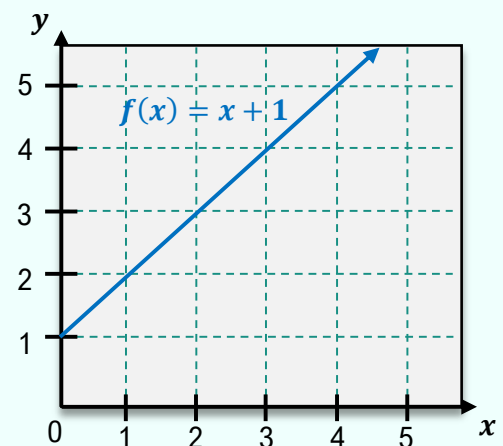
- ◆ Recall: The more subintervals (n) used, the more accurate the Riemann sum estimate for the area under a curve.
- Taking the limit as n approaches _____ gives the *true* area under the curve. This is called the **definite integral**.



EXAMPLE

Express the following limit as a definite integral on the interval $[0,4]$ and find the *exact* area that it represents.

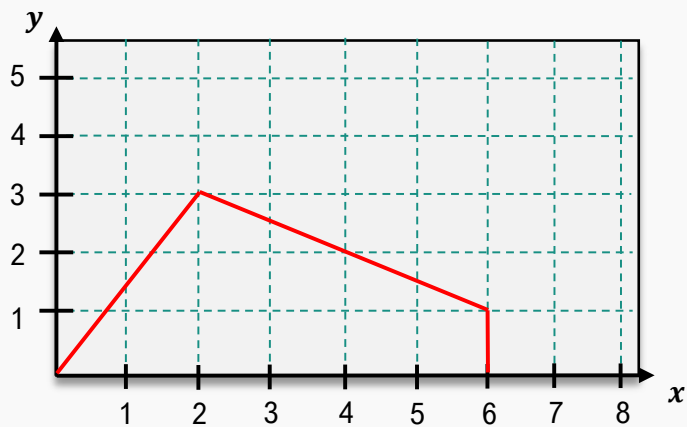
$$\lim_{n \rightarrow \infty} \sum_{k=1}^n (x_k^* + 1) \Delta x$$



TOPIC: INTRODUCTION TO THE DEFINITE INTEGRAL

PRACTICE

Find $\int_0^6 f(x) dx$ given the graph of $y = f(x)$.



PRACTICE

Express the following limit as a definite integral on the interval $[0, 10]$.

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n (x_k^* - 3)^2 \Delta x$$

TOPIC: INTRODUCTION TO THE DEFINITE INTEGRAL

Basic Rules for Definite Integrals

◆ The basic rules that apply to indefinite integrals also apply to definite integrals.

► Definite integrals have *additional* rules related to their _____.

RULES OF INTEGRATION		
Name	Rule	Example
Sum & Difference	$\int_a^b [f(x) \pm g(x)] dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$	$\int_{-2}^5 (x + 6) dx = \int_{-2}^5 x dx + \int_{-2}^5 6 dx$
Constant Multiple	$\int_a^b k f(x) dx = k \int_a^b f(x) dx$	$\int_2^6 5x^3 dx = 5 \int_2^6 x^3 dx$
Order of Integration	$\int_b^a f(x) dx = - \int_a^b f(x) dx$	$\int_4^1 x^3 dx = - \int_1^4 x^3 dx$
Zero Width Interval	$\int_a^a f(x) dx = 0, \text{ if } f(a) \text{ exists}$	$\int_3^3 (3x^2 - 4x) dx = 0$
Additivity	$\int_a^c f(x) dx + \int_c^b f(x) dx = \int_a^b f(x) dx, \quad a < c < b$	$\int_1^3 5x^4 dx + \int_3^7 5x^4 dx = \int_1^7 5x^4 dx$

◆ Use *multiple* rules together to find integrals of more complicated functions.

TOPIC: INTRODUCTION TO THE DEFINITE INTEGRAL

PRACTICE

Given the following definite integral of the function $f(x) = 3x^2 - 2x$, write the simplified integral $-\int_4^0 f(x) dx$.

PRACTICE

Write the two definite integrals subtracted below as a single integral.

$$\int_1^6 \sqrt{x^2 - 5x} \, dx - \int_{10}^6 \sqrt{x^2 - 5x} \, dx$$

TOPIC: INTRODUCTION TO THE DEFINITE INTEGRAL

PRACTICE

Evaluate the following definite integral.

$$\int_{100}^{100} \frac{\sin(2x) - 100x^2}{\sqrt{x^5 + \tan(3x - 6)}} dx$$