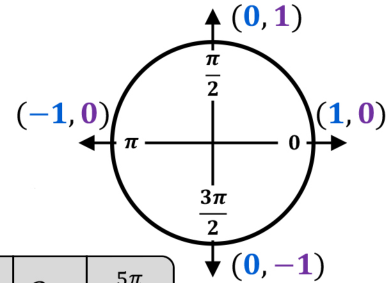
# TOPIC: GRAPHS OF SINE & COSINE

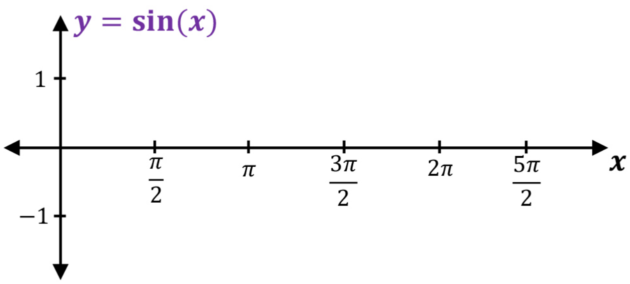
## Graphing Sine & Cosine (with Vertical Shifts)

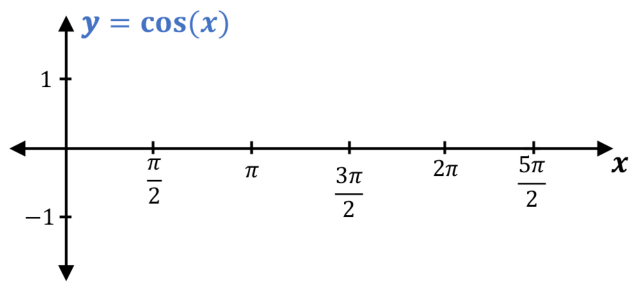



- Sine & Cosine values _________ around the unit circle, so their graphs are _________.
  - The high points are "crests" or ________; the low points are "troughs" or ________.

| $x$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ | $\frac{5\pi}{2}$ |
|---|---|---|---|---|---|---|
| $\sin x$ | | | | | | |

| $x$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ | $\frac{5\pi}{2}$ |
|---|---|---|---|---|---|---|
| $\cos x$ | | | | | | |




- Just as we've vertically shifted functions, we can also vertically shift $\sin$ & $\cos$ by adding a constant ___.
  - For positive $k$, graph shifts **[UP | DOWN]**; for negative $k$, graph shifts **[UP | DOWN]**.

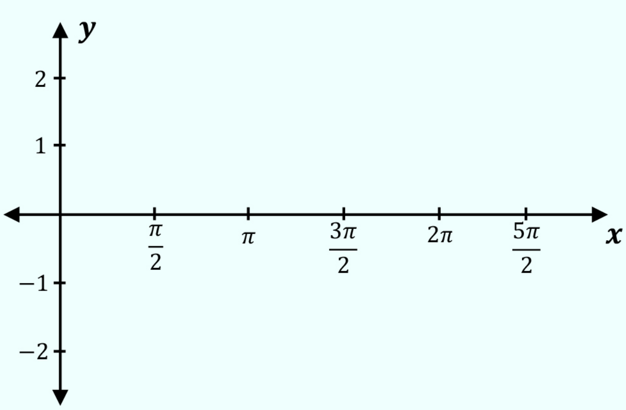
$y = \sin(x) + \_\_$

$y = \cos(x) + \_\_$

**Recall** **Transformations**

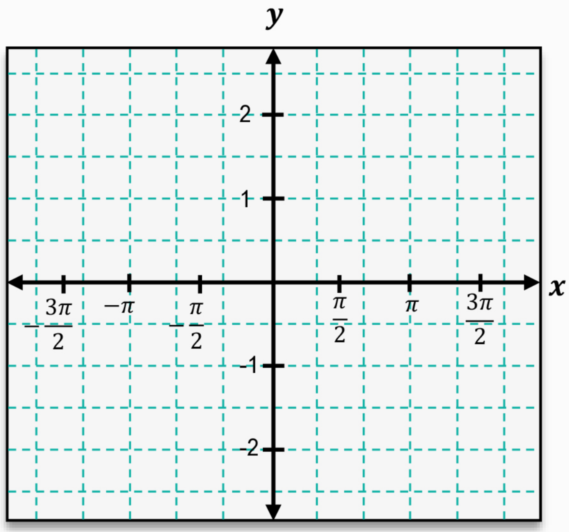
$g(x) = f(x) + k$

**EXAMPLE** Graph the function $y = \sin(x) + 1$.

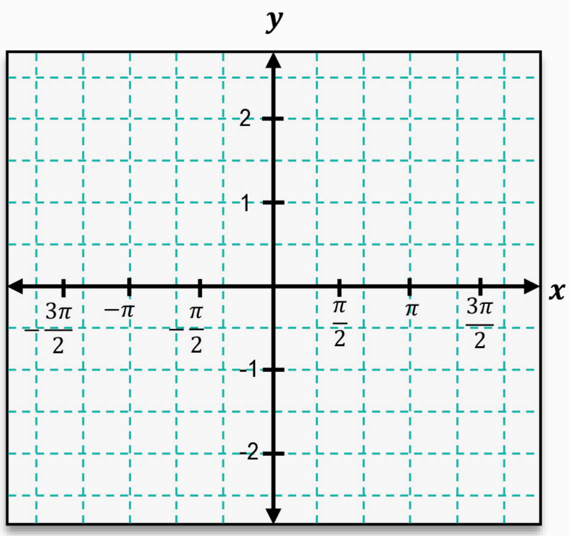


## TOPIC: GRAPHS OF SINE & COSINE

**PRACTICE** Sketch the function $y = \cos(x) - 1$ on the graph below.

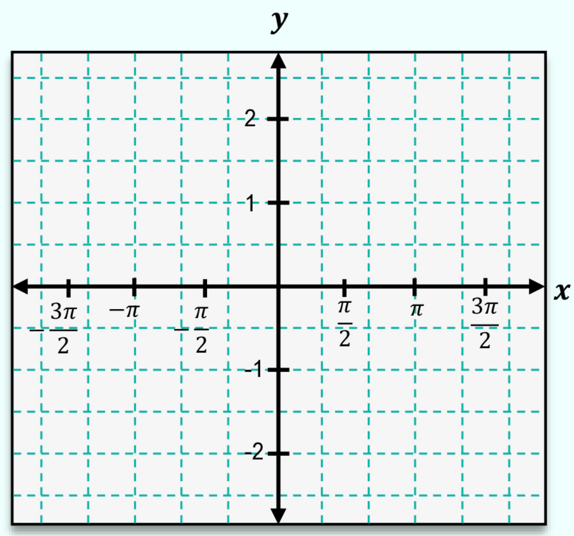



**PRACTICE** Determine the value of $y = \sin\left(-\frac{\pi}{2}\right) + 50$ without using a calculator or the unit circle.



## TOPIC: GRAPHS OF SINE & COSINE

**EXAMPLE** Graph the function $y = \sin(x) + 3$.

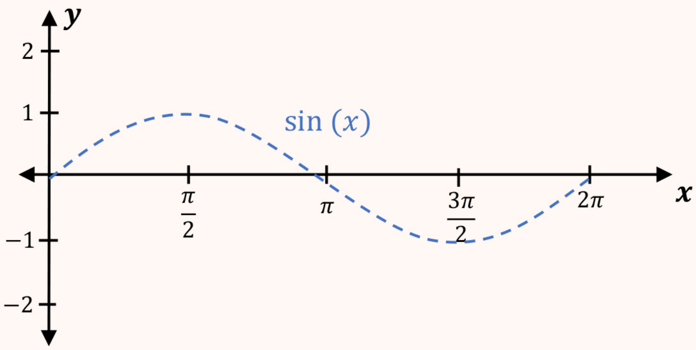


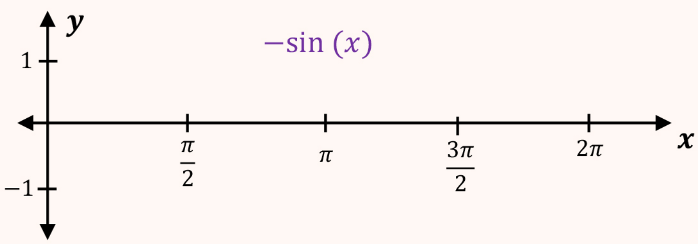
## TOPIC: GRAPHS OF SINE & COSINE

### Amplitude & Reflection of Sine & Cosine

- Recall: Sine & Cosine graphs are repeating waves.
  - **Amplitude:** # affecting how ______ the peaks are, i.e. the distance from the midline to the peaks or valleys.
  - If the amplitude is negative, the graph is _____________ over the x-axis.

New **Amplitude**



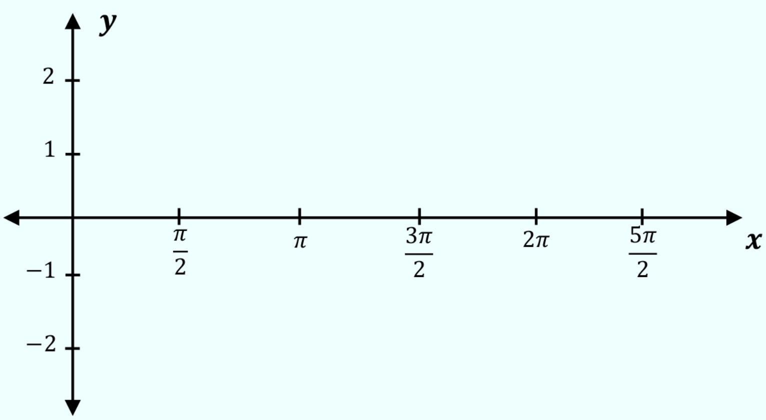




| $x$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
|---|---|---|---|---|---|
| $\sin x$ | $0$ | $1$ | $0$ | $-1$ | $0$ |
| $2 \cdot \sin x$ | | | | | |
| $-\sin x$ | | | | | |

$y = \_\_ \sin(x) + k$

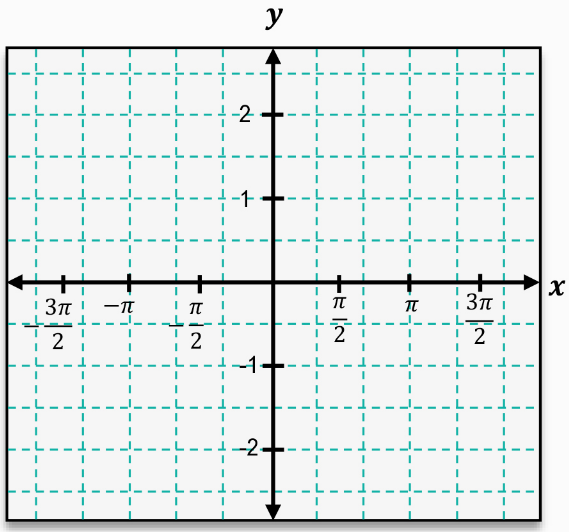
$y = \_\_ \cos(x) + k$

**EXAMPLE** Graph the function $y = -\frac{3}{2} \cdot \cos x$

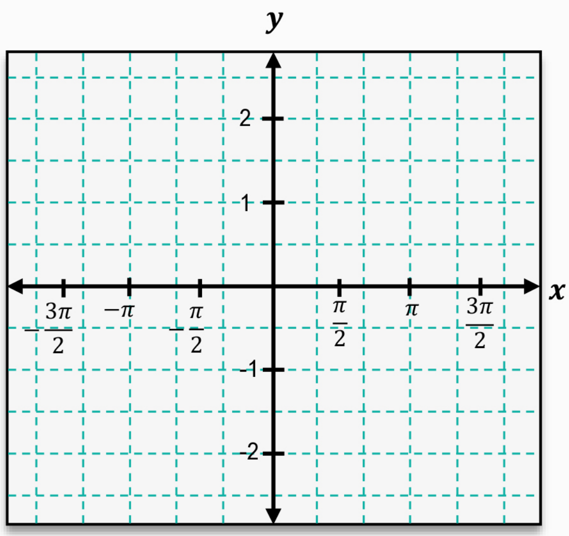


## TOPIC: GRAPHS OF SINE & COSINE

**PRACTICE** Determine the value of $y = -2 \cdot \sin\left(-\frac{3\pi}{2}\right) + 10$ without using a calculator or the unit circle.

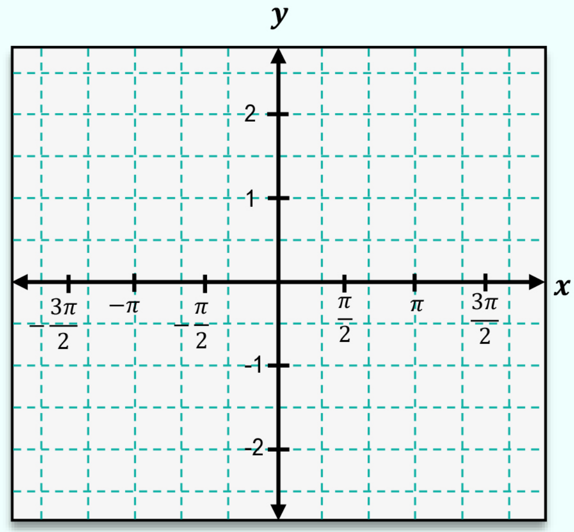



**PRACTICE** Graph the function $y = -3 \cdot \cos(x)$.



## TOPIC: GRAPHS OF SINE & COSINE

**EXAMPLE** Graph the function $y = 2 \cdot \sin(x) - 1$.

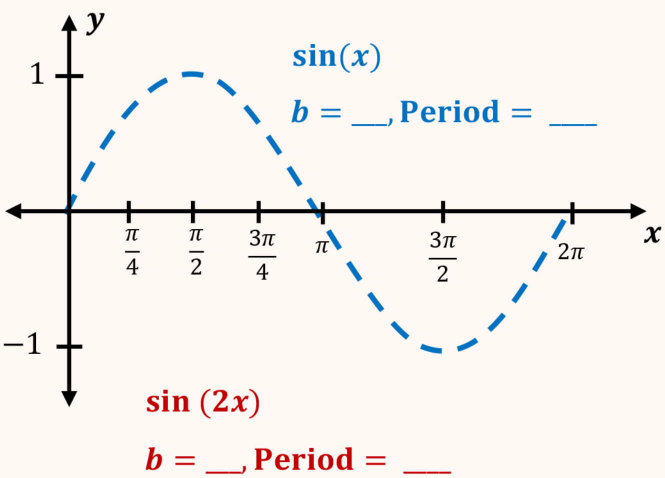


# TOPIC: GRAPHS OF SINE & COSINE

## Period of Sine & Cosine

- **Period:** How "______" the graph is, i.e. the distance (along $x$) of a full wave or cycle.
  - Period is modified by a # (given by ___) in front of $x$ in $\sin(x)$ or $\cos(x)$.

**New** — **Period of Sine & Cosine Graphs**




| $x$ | $0$ | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ | $\frac{5\pi}{4}$ | $\frac{3\pi}{2}$ | $\frac{7\pi}{4}$ |
|---|---|---|---|---|---|---|---|---|
| $\sin(x)$ | $0$ | $\frac{\sqrt{2}}{2}$ | $1$ | $\frac{\sqrt{2}}{2}$ | $0$ | $\frac{-\sqrt{2}}{2}$ | $-1$ | $\frac{-\sqrt{2}}{2}$ |
| $\sin(2x)$ | | | | | | 1 | 0 | -1 |

$y = \mathrm{A}\sin(\_\_\_x) + k$

$y = \mathrm{A}\cos(\_\_\_x) + k$

Period $(T) = \frac{\quad}{\quad}$

For $b > 1$, graph horizontally **[SHRINKS | STRETCHES]**

For $0 < b < 1$, graph horizontally **[SHRINKS | STRETCHES]**

**EXAMPLE** Without graphing, calculate the period of the following functions.

**(A)**

$$y = \sin\left(\frac{1}{2}x\right)$$

**(B)**

$$y = \cos(4\pi x)$$

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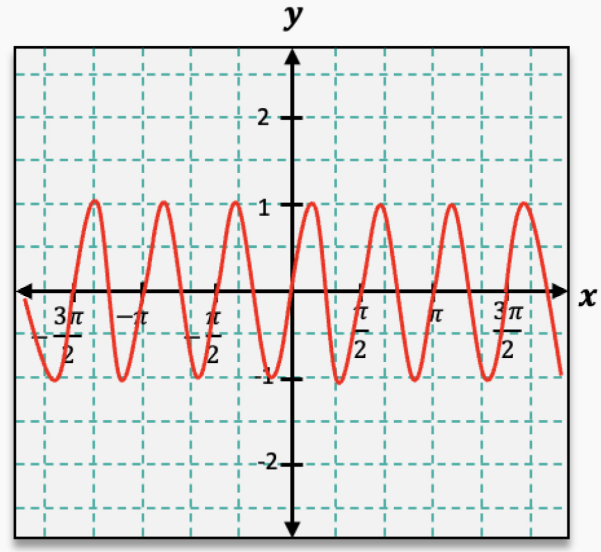
**PRACTICE** Given below is the graph of the function $y = \sin(bx)$. Determine the correct value for $b$.




**PRACTICE** The period for the function $y = \cos(bx)$ is $T = 20\pi$. Determine the correct value for $b$.

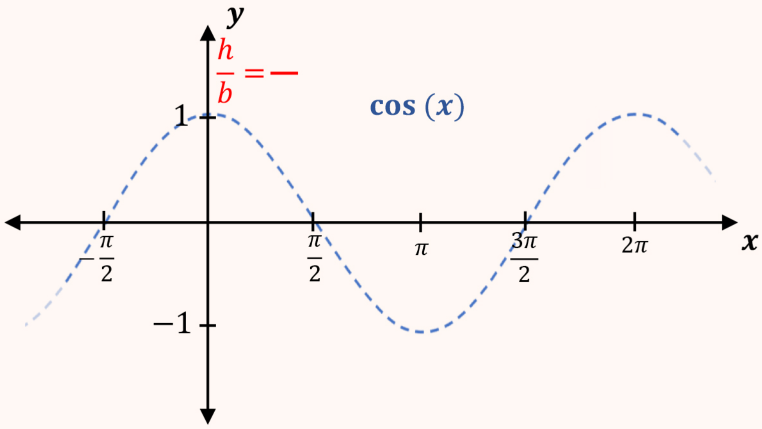
# TOPIC: GRAPHS OF SINE & COSINE

## Phase Shifts

◆ **Phase shift** ($\boldsymbol{h}$): a ___________________ shift (left/right), indicated by numbers **[INSIDE | OUTSIDE]** parentheses.

- A phase shift can make a *cosine* graph look like a ______ graph.

**New** — **Phase Shifts of Sine & Cosine Graphs**




| $x$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
|---|---|---|---|---|---|
| $\cos x$ | $1$ | $0$ | $-1$ | $0$ | $1$ |
| $\cos\left(x-\frac{\pi}{2}\right)$ | | | | | |

$y = A\sin\ (bx - \underline{\quad}^{*}) + k$

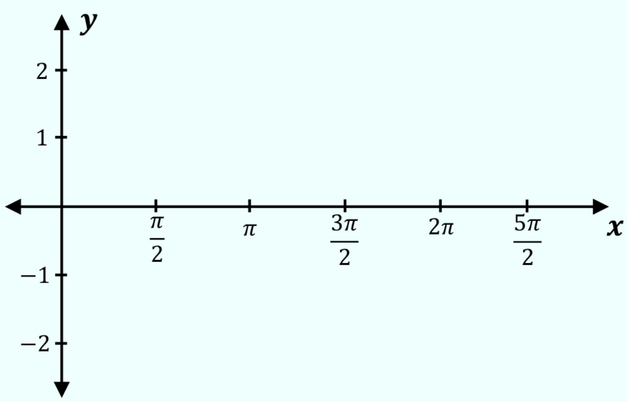
$y = A\cos\ (bx - \underline{\quad}^{*}) + k$

If given $(bx - \#)$ $h$ is positive and graph shifts **[← | →]** by $\frac{h}{b}$

If given $(bx + \#)$ $h$ is negative and graph shifts **[← | →]** by $\frac{h}{b}$

*you may see other letters for this $(\phi, C, d)$

**EXAMPLE** Graph $y = \sin\ (2x + \pi)$ over one full period.



## TOPIC: GRAPHS OF SINE & COSINE

**PRACTICE** Describe the phase shift for the following function: $y = \cos\left(5x - \frac{\pi}{2}\right)$

**PRACTICE** Describe the phase shift for the following function: $y = \cos\left(2x + \frac{\pi}{6}\right)$

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**EXAMPLE** Graph the function $y = 3 \cdot \sin(x + \pi)$.