MASTER TABLE: TRIGONOMETRIC IDENTITIES

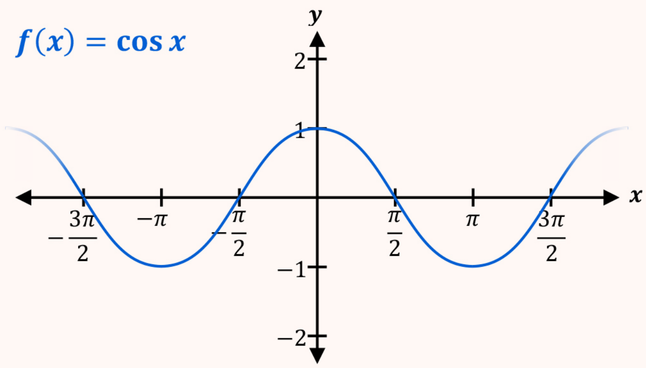
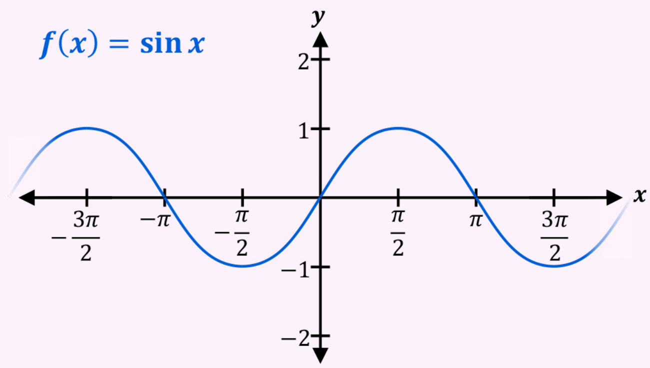
◆ **NOTE**: This table spans multiple videos.

TRIG IDENTITIES			
Name	**Identity**	**Example**	**Use when...**
Reciprocal	$\csc\theta = \frac{1}{\sin\theta}$	$\sec\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\left(\frac{1}{2}\right)} = 2$	You need to rewrite an expression in terms of $\sin$ & $\cos$
	$\sec\theta = \frac{1}{\cos\theta}$		
	$\cot\theta = \frac{1}{\tan\theta}$		
Quotient	$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\tan\frac{\pi}{4} = \frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)}$	
	$\cot\theta = \frac{\cos\theta}{\sin\theta}$		
Even – Odd	$\cos(-\theta) = \_\_\_\cos\theta$	$\cos\left(-\frac{\pi}{4}\right) =$	*argument* is ______________.
	$\sin(-\theta) = \_\_\_\sin\theta$	$\csc\left(\frac{\pi}{6}\right) =$	
	$\tan(-\theta) = \_\_\_\tan\theta$		
Pythagorean	$\sin^2\theta + \cos^2\theta = 1$	$\sin^2\frac{11\pi}{6} + \cos^2\frac{11\pi}{6} =$	you see trig functions _________.
	$\_\_\_\_\theta + \_\_\_\_\_ = \_\_\_\_\theta$		
	$\_\_\_\_\_ + \_\_\_\_\theta = \_\_\_\_\theta$		
Sum & Diff.	$\sin(a \pm b) = \_\_\_\_a\ \_\_\_\_\_b \pm \_\_\_\_\_a\ \_\_\_\_\_b$	$\sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right) =$	argument contains a _____/_____ *OR* multiples of $15°$ or $\frac{\pi}{12}$
	$\cos(a \pm b) = \_\_\_\_a\ \_\_\_\_b \mp \_\_\_\_\_a\ \_\_\_\_\_b$		
	$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$		
Double Angle	$\sin 2\theta = 2\_\_\_\_\_\_\_\_\_\_\_\_\_\_$	$\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12} =$	argument contains ______ *OR* you recognize a _______ of the identity
	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $= 1 - 2\sin^2\theta$ $= 2\cos^2\theta - 1$		
	$\tan 2\theta = \frac{2\tan\theta}{1 - \_\_\_\_\_^2\theta}$		

TOPIC: TRIGONOMETRIC IDENTITIES

Even & Odd Identities

◆ If you know a function is **even** or **odd**, you can easily find $f($ ______ $)$.

Even Function	Odd Function
	
$f(-x) =$ [$f(x)$ | $-f(x)$] Symmetric on [y – **axis** | **ORIGIN**] Trig Functions: [**cos sec** | **sin csc** | **tan cot**]	$f(-x) =$ [$f(x)$ | $-f(x)$] Symmetric on [y – **axis** | **ORIGIN**] Trig Functions: [**cos sec** | **sin csc** | **tan cot**]

◆ An **identity** is an equation which is TRUE for ________ possible values.

TRIG IDENTITIES

Name	Identity	Example	Use when…
Even – Odd	$\cos(-\theta) =$ ____ $\cos\theta$	$\cos\left(-\frac{\pi}{4}\right) =$	*argument* is ____________.
	$\sin(-\theta) =$ ____ $\sin\theta$	$\csc\left(-\frac{\pi}{6}\right) =$	
	$\tan(-\theta) =$ ____ $\tan\theta$		

TOPIC: TRIGONOMETRIC IDENTITIES

EXAMPLE Use the even/odd identities to rewrite the expression with no negative arguments in terms of one trig function.

(A) $-\tan(-\theta)$

(B) $\dfrac{\sin(-\theta)}{\cos(-\theta)}$

Recall

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

(*Even – Odd Identities*)

PRACTICE Use the even-odd identities to evaluate the expression.

(A) $\cos(-\theta) - \cos\theta$

(B) $-\cot(\theta) \cdot \sin(-\theta)$

TOPIC: TRIGONOMETRIC IDENTITIES

PRACTICE Use the even-odd identities to evaluate the expression.

Recall

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

(Even – Odd Identities)

(A)

$$\sec\left(-\frac{4\pi}{5}\right)$$

$\cos\frac{4\pi}{5}$ $-\cos\frac{4\pi}{5}$ $\sec\frac{4\pi}{5}$ $-\sec\frac{4\pi}{5}$

(B)

$$\sin(-38°)$$

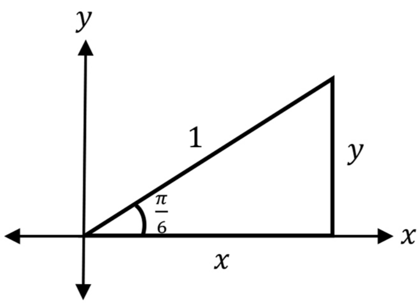
$\sin 38°$ $-\sin 38°$ $-\sin(-38°)$ $\dfrac{1}{-\sin 38°}$

TOPIC: TRIGONOMETRIC IDENTITIES

Pythagorean Identities

◆ You'll need the **Pythagorean Identities** to simplify expressions with _____________ trig functions.

- These identities come from combining the Pythagorean Theorem with the Unit Circle.



$$a^2 + b^2 = c^2$$

$$y^2 + x^2 = 1$$

TRIG IDENTITIES			
Name	**Identity**	**Example**	**Use when…**
Pythagorean	$\sin^2\theta + \cos^2\theta = 1$	$\sin^2\frac{11\pi}{6} + \cos^2\frac{11\pi}{6} =$	you see trig functions **squared**.
	______ θ + ____ = _____ θ		
	_______ + ____ θ = _____ θ		

◆ To rewrite trig expressions, you'll need to recognize different _________ of the Pythagorean Identities.

EXAMPLE Use the Pythagorean Identities to rewrite the expression as a single term.

(A)

$\sec^2\theta - \tan^2\theta$

(B)

$(1 - \cos\theta)(1 + \cos\theta)$

TOPIC: TRIGONOMETRIC IDENTITIES

EXAMPLE Use the Pythagorean Identities to rewrite the expression as a single term.

$1 + \cos\theta$

Recall

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

(*Even – Odd Identities*)

PRACTICE Use the Pythagorean Identities to rewrite the expression as a single term.

$(1 + \csc\theta)(1 - \csc\theta)$

PRACTICE Use the Pythagorean Identities to rewrite the expression with no fraction.

$$\frac{1}{1 - \sec\theta}$$

TOPIC: TRIGONOMETRIC IDENTITIES

Sum and Difference of Sine & Cosine

◆ The sum & difference identities are useful when you have multiple angles in the argument of a trig function.

TRIG IDENTITIES			
Name	**Identity**	**Example**	**Use when...**
Sum & Difference	$\sin(a+b) =$ ____ $\boldsymbol{a}$ ____ $\boldsymbol{b}$ ____ ____ $\boldsymbol{a}$ ____ $\boldsymbol{b}$	$\sin\left(\frac{\pi}{2}+\frac{\pi}{6}\right)$ OR $\sin(90°+30°)$	argument contains a ____/____ *OR* multiples of $15°$ or $\frac{\pi}{12}$
	$\sin(a-b) = \sin\boldsymbol{a}\ \cos\boldsymbol{b}\quad \cos\boldsymbol{a}\ \sin\boldsymbol{b}$		
	$\cos(a+b) =$ ____ $\boldsymbol{a}$ ____ $\boldsymbol{b}$ ____ ____ $\boldsymbol{a}$ ____ $\boldsymbol{b}$		
	$\cos(a-b) = \cos\boldsymbol{a}\ \cos\boldsymbol{b}\quad \sin\boldsymbol{a}\ \sin\boldsymbol{b}$		

◆ To find exact values of trig functions NOT on the unit circle, rewrite argument as sum or diff. of 2 KNOWN angles.

EXAMPLE Find the exact value of the function.

$\cos 15°$

TOPIC: TRIGONOMETRIC IDENTITIES

EXAMPLE Rewrite each argument as the sum or difference of two angles on the unit circle.

(A) $\sin(75°)$

(B) $\cos(-15°)$

(C) $\cos\left(\frac{7\pi}{12}\right)$

PRACTICE Find the exact value of the expression.

(A) $\cos 105°$

Recall

$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

(Sum/Difference Identities)

(B) $\sin 15°$

(C) $\cos\frac{5\pi}{12}$

TOPIC: TRIGONOMETRIC IDENTITIES

EXAMPLE Find the exact value of the expression.

$\sin 10° \cos 20° + \sin 20° \cos 10°$

Recall

$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

(*Sum/Difference Identities*)

PRACTICE Find the exact value of the expression.

$\cos 80° \cos 20° + \sin 80° \sin 20°$

TOPIC: TRIGONOMETRIC IDENTITIES

EXAMPLE Expand the expression using the sum & difference identities and simplify.

Recall

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

(Sum/Difference Identities)

$(\boldsymbol{A})$

$\sin(\theta + 30°)$

$(\boldsymbol{B})$

$\cos\left(\frac{\pi}{4} - \theta\right)$

PRACTICE Expand the expression using the sum & difference identities and simplify.

$\sin\left(-\theta - \frac{\pi}{2}\right)$

TOPIC: TRIGONOMETRIC IDENTITIES

Double Angle Identities

◆ When the sum identities are used with the *same* two angles, we get the **double angle identities**.

TRIG IDENTITIES			
Name	**Identity**	**Example**	**Use when…**
Sum & Diff.	$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$ $\sin(\theta + \theta) = \sin\theta\cos\theta + \cos\theta\sin\theta$		
Double Angle	$\sin 2\theta = 2$________________	$\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12} =$	argument contains ______ *OR* you recognize a _______ of the identity
	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $= 1 - 2\sin^2\theta$ $= 2\cos^2\theta - 1$		
	$\tan 2\theta = \dfrac{2\tan\theta}{1 - \_\_\_\_\_^2\,\theta}$		

◆ To simplify expressions & verify identities, you'll need to recognize different **forms** of the Double Angle Identities.

EXAMPLE Simplify the expression, do not evaluate.

$\sin 15° \cos 15°$