

TOPIC: RIEMANN SUMS

Sigma Notation

- ◆ We can write a sum involving many terms in a more _____ way using **sigma notation**.

New

Sigma Notation

The diagram illustrates the components of the sigma notation $\sum_{k=1}^n a_k$. It shows the summation symbol $\sum$ with a superscript n and a subscript $k=1$. The term a_k follows. An arrow points from the text "index _____ here" to the superscript n . Another arrow points from the text "formula for the k^{th} term" to the term a_k . A third arrow points from the text "index _____ here" to the subscript $k=1$. A fourth arrow points from the text "_____ of summation" to the subscript $k=1$. The equation is followed by an equals sign and the expanded sum $a_1 + a_2 + \dots + a_n$, where the subscript 1 is cyan and the superscript n is purple.

- ◆ To evaluate a sum, plug in integer values, from the **start index** to the **end index**, into a_k , adding all terms together.
- The index of summation (k) can be represented by *any* letter and can start at *any* integer.

EXAMPLE

Evaluate the following finite sums.

(A)

$$\sum_{k=1}^3 k^2$$

(B)

$$\sum_{i=0}^4 i + 3$$

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PRACTICE

Evaluate the following summation:

(A)

$$\sum_{i=1}^4 i^2 - 3i + 8$$

(B)

$$\sum_{k=1}^4 \left(\frac{k}{2}\right)^2$$

(C)

$$\sum_{i=0}^2 \frac{2i}{3}$$

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EXAMPLE

Write the following expanded sums in sigma notation:

(A) $3 \cdot 1^2 + 3 \cdot 2^2 + 3 \cdot 3^2 + 3 \cdot 4^2 + 3 \cdot 5^2$

(B) $\frac{3}{4} + \frac{3}{2} + \frac{9}{4} + 3$

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Algebra Rules for Finite Sums

◆ Just like for derivatives and indefinite integrals, there are sum/difference and constant rules for sums.

RULES FOR FINITE SUMS		
Name	Rule	Example
Sum & Difference	$\sum_{k=1}^n (a_k \pm b_k) = \sum_{k=1}^n a_k \pm \sum_{k=1}^n b_k$	$\sum_{k=1}^3 (k^2 - k) = \sum_{k=1}^3 \text{---} - \sum_{k=1}^3 \text{---} =$
Constant Multiple	$\sum_{k=1}^n c \cdot a_k = \text{---} \cdot \sum_{k=1}^n a_k$	$\sum_{k=1}^3 2k^2 = \text{---} \cdot \sum_{k=1}^3 \text{---} =$
Constant Value	$\sum_{k=1}^n c = c \cdot \text{---}$	$\sum_{k=1}^{18} 3 = \text{---} \cdot \text{---} = \text{---}$

◆ Use *multiple* rules together to find summations of more complicated functions.

EXAMPLE Evaluate the given sum.

$$\sum_{k=1}^3 (6k^2 - 9)$$

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PRACTICE

Evaluate the following summations:

(A)

$$\sum_{t=2}^6 5t + \frac{4}{t}$$

(B)

$$\sum_{i=0}^4 2i + 1$$

(C)

$$\sum_{i=1}^2 4i^3 - 3i^2 + 2i - 1$$

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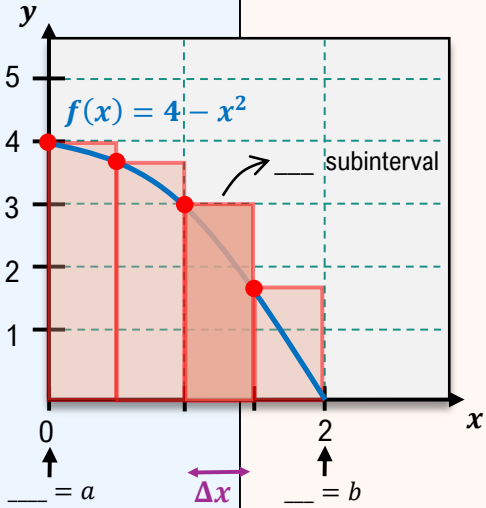
Introduction to Riemann Sums

◆ Recall: We can estimate the area under the curve by breaking it into n rectangles (subintervals) of width $\Delta x = \frac{b-a}{n}$.

► We can use _____ notation to represent the estimated area as a **Riemann sum**.

EXAMPLE

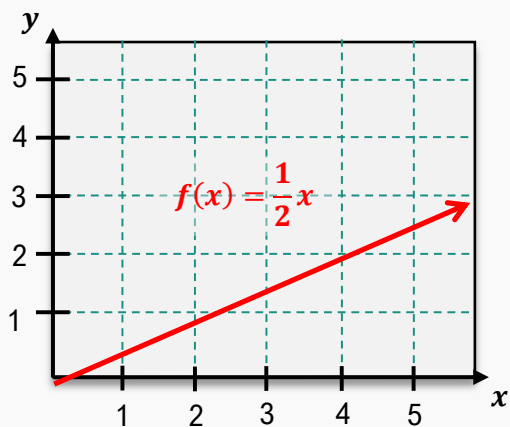
Estimate the area under the curve using a left endpoint Riemann sum with 4 subintervals.

Recall	Estimating Area	New	Estimating Area with Riemann Sums
	$A \approx A_1 + A_2 + A_3 + A_4$ $\approx w_1 h_1 + w_2 h_2 + w_3 h_3 + w_4 h_4$ $\approx \Delta x (f(0) + f(0.5) + f(1) + f(1.5))$ $\approx (0.5)(4 + 3.75 + 3 + 1.75)$ ≈ 6.25	 $\text{---} = a$ Δx $\text{---} = b$	$A \approx \sum_{k=1}^n f(\text{---}) \Delta x$ (Left Riemann sum) $A \approx \sum_{k=1}^n \Delta x$ $A \approx \text{---} \sum_{k=1}^n$ $\approx (\Delta x)$ ≈ 6.25 $A \approx \sum_{k=1}^n f(\text{---}) \Delta x$ (General Riemann sum)

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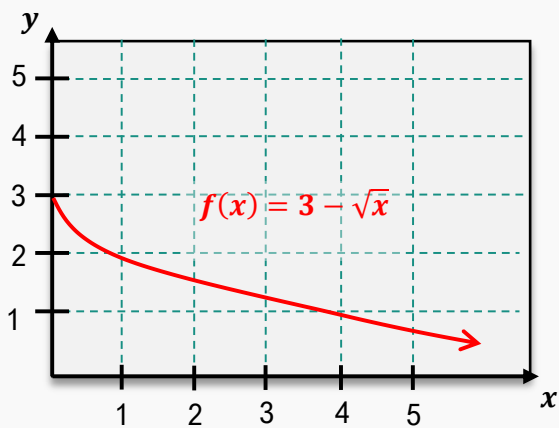
PRACTICE

For the following graph, write a Riemann sum using left endpoints to approximate the area under the curve over $[0,5]$ with 5 subintervals.



PRACTICE

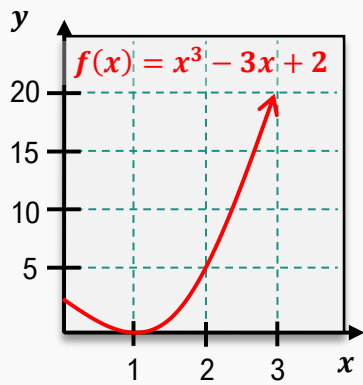
For the following graph, write a Riemann sum using left endpoints to approximate the area under the curve over $[0,6]$ with 6 subintervals.



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PRACTICE

Write the Riemann sum that would approximate the area of the following graph over the interval $[0,3]$ using 3 subintervals.



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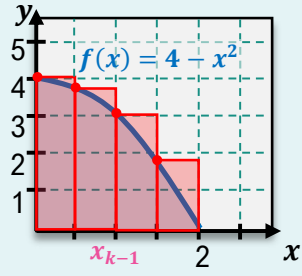
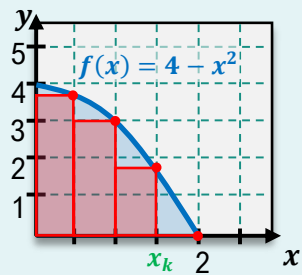
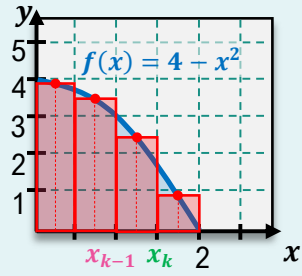
Left, Right, & Midpoint Riemann Sums

◆ Recall: When estimating area under a curve, we can use fcn values at the left endpoint, right endpoint, or midpoint.

► For Riemann sums, we can do the same thing.

EXAMPLE

Set up the left, right, and midpoint Riemann sums for $f(x) = 4 - x^2$ on $[0, 2]$.

Left, Right, & Midpoint Riemann Sums		
Left	$x_k^* = x_{k-1}$ is the left endpoint of subinterval $x_{k-1} = a + \underline{\hspace{1cm}} \Delta x$ Recall $A \approx L_n = \sum_{k=1}^n f(x_{k-1}) \Delta x$ (Left Riemann Sum)	 $L_n = \sum_{k=1}^n (\underline{\hspace{1cm}})(\underline{\hspace{1cm}})$
Right	$x_k^* = \underline{\hspace{1cm}}$ is the right endpoint of subinterval $x_k = a + \underline{\hspace{1cm}} \Delta x$ New $A \approx R_n = \sum_{k=1}^n f(\underline{\hspace{1cm}}) \Delta x$ (Right Riemann sum)	 $R_n = \sum_{k=1}^n (\underline{\hspace{1cm}})(\underline{\hspace{1cm}})$
Midpoint	$x_k^* = \underline{\hspace{1cm}}$ is the midpoint of subinterval $x_k^* = a + \underline{\hspace{1cm}} \Delta x$ New $A \approx M_n = \sum_{k=1}^n f(\underline{\hspace{1cm}}) \Delta x$ (Midpoint Riemann sum)	 $M_n = \sum_{k=1}^n (\underline{\hspace{1cm}})(\underline{\hspace{1cm}})$

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PRACTICE

Approximate the area under the curve $f(x) = \sqrt{x+3}$ over the interval $[1,5]$ using the Right Riemann sum with 8 subintervals.

PRACTICE

Approximate the area under the curve $f(x) = \frac{2}{3x+5}$ over the interval $[4,10]$ using the Midpoint Riemann sum with $n = 3$ subintervals.

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EXAMPLE

Set up and solve the left, right, and midpoint Riemann sums for $f(x) = x^3$ on the interval $[0,4]$. Use 4 subintervals. Tell which is an overestimate, and which is an underestimate.