

TOPIC: BASICS OF DIFFERENTIAL EQUATIONS

Classifying Differential Equations

◆ A differential equation (DE) is an equation involving a function $y = f(x)$ and its _____.

► A DE can be classified by its order (_____ derivative) and whether it is linear or nonlinear.

EXAMPLE

Determine the order of the following differential equations and indicate if they are linear.

(A)

$$y'' + 4y' - 3y = \sin(e^x)$$

Order: ____

Dependent var. (often y) & its derivatives are...

Not multiplied by each other ☐

Only raised to 1st power ☐

Not the "inside" of a function ☐

[LINEAR | NONLINEAR]

(B)

$$ty'' - t^2y^{(4)} = e^y$$

Order: ____

Dependent var. (often y) & its derivatives are...

Not multiplied by each other ☐

Only raised to 1st power ☐

Not the "inside" of a function ☐

[LINEAR | NONLINEAR]

(C)

$$y \cdot \frac{dy}{dx} + 5 \left(\frac{dy}{dx} \right)^3 - 4 = 2x$$

Order: ____

Dependent var. (often y) & its derivatives are...

Not multiplied by each other ☐

Only raised to 1st power ☐

Not the "inside" of a function ☐

[LINEAR | NONLINEAR]

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PRACTICE

State the order of the differential equation and indicate if it is linear or nonlinear.

(A) $y''' + 3xy = 4\sqrt{x}$

(B) $\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)(1-x) = 2$

(C) $(y'')^2 + 6e^t y' = 4t$

(D) $y' \cdot y = 3e^t$

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Verifying Solutions of Differential Equations

◆ The solution of a DE is any function $y = f(x)$ that makes the equation true when y & its *derivatives* are plugged in.

EXAMPLE

Verify that the following functions are solutions to the given differential equations.

(A)

$$y = e^{2x}; \quad 3y' - 5y = e^{2x}$$

(B)

$$y = 4 + \ln x; \quad xy'' + y' = 0$$

◆ DEs can also be in *implicit* form (y is not isolated) which requires *implicit differentiation* of the solution to verify.

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EXAMPLE

Use implicit differentiation to show that the given solution satisfies the indicated differential equation.

Solution: $4x^2 = 2y^3 + 4y$; DE: $(3y^2 + 2)y' = 4x$

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EXAMPLE

Determine if $y = e^{4x}$ is a solution to the differential equation $y'' - 4y' + 3y = 0$.

EXAMPLE

Show that the given function is the general solution of the indicated differential equation. Assume that C is an arbitrary constant.

Solution: $y = \frac{C}{x^2}$

DE: $xy' = -2y$

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EXAMPLE

Verify that the given function is a solution of the differential equation that follows it. Assume C is an arbitrary constant.

$$y(t) = Ce^{-6t}; \quad y'(t) + 6y(t) = 0$$

EXAMPLE

Use implicit differentiation to show that the given solution satisfies the indicated differential equation. Assume C is an arbitrary constant.

Solution: $y + x^2y^2 - 3x = C$

DE: $(1 + 2x^2y)y' = 3 - 2xy^2$

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Solutions to Basic Differential Equations

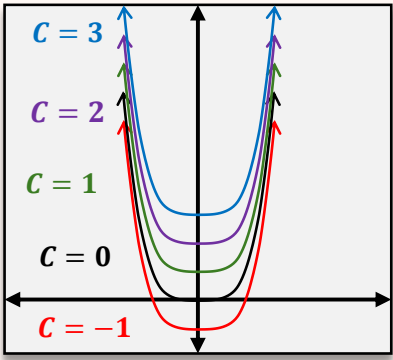
- ◆ The *general* solution of a basic DE is the antideriv. w/ integration constant C , which represents a _____ of fcn's.
- ▶ The *particular* solution of a DE is found by using an initial condition (a _____ on the function curve).

EXAMPLE

Find the particular solution to the differential equation $y' = 4x^3$ passing through the point $(1,4)$.

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Particular Solution of a Differential Equation



$$= \int \quad dx$$

$$y =$$

Find general solution

$$= \quad + C$$

Apply initial condition

$$C =$$

$$y =$$

Write particular solution

EXAMPLE

Verify that $y = Ce^{x^2}$ is a solution to the differential equation $y' = 2xy$ and find the particular solution that passes through the point $(0,6)$.

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PRACTICE

Find the particular solution to the differential equation $y' = 2e^t + 4t$ given the initial condition $y(0) = 1$.

PRACTICE

Find the general solution to the differential equation $\frac{dy}{dx} = -2x + 5x^2$.