

TOPIC: HYPERGEOMETRIC DISTRIBUTION



Intro to Hypergeometric Distribution

◆ Recall: **Binomial** Variable: $X = \#$ of successes out of n trials, with constant $P(\text{success}) = p$.

► The **Hypergeometric** Variable: $X = \#$ of occurrences in n draws from a group of N items *without* _____.

EXAMPLE

You draw 3 marbles from a bag containing 2 red & 4 blue. Find the probability that exactly 1 of the marbles you pick is red if you are drawing...

Recall	Binomial Distribution	New	Hypergeometric Distribution
	<p>(A) <i>with</i> replacement</p> <p>Trial = draw marble</p> <p>Success = red</p> <p><input type="checkbox"/> Only 2 outcomes?</p> <p><input type="checkbox"/> Fixed # of trials?</p> <p><input type="checkbox"/> Independent trials?</p> <p><input type="checkbox"/> Equal $P(\text{success})$ per trial?</p> <p>$n = 3, \quad p = \frac{2}{6}, \quad q = \frac{4}{6}$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $P(x) = \binom{n}{x} \cdot p^x \cdot q^{n-x}$ </div> $P(X = 1) = \binom{3}{1} \cdot \left(\frac{2}{6}\right)^1 \cdot \left(\frac{4}{6}\right)^{3-1}$ $= 3 \cdot \left(\frac{1}{3}\right) \cdot \left(\frac{2}{3}\right)^2 = \frac{4}{9}$ 		<p>(B) <i>without</i> replacement</p> <p>Trial = draw marble</p> <p>Success = red</p> <p><input type="checkbox"/> Only 2 outcomes?</p> <p><input type="checkbox"/> Fixed # of trials?</p> <p><input type="checkbox"/> Selecting w/o replacement?</p> <p>$n = \#$ of draws = _____</p> <p>$r = \#$ of successes in group = _____</p> <p>$N = \#$ of total items to select from = _____</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $P(x) = \frac{\# \text{ draws with 1 red}}{\# \text{ draws possible}} = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}}$ </div> 

PRACTICE

A school is holding a fair raffle and a teacher is interested in predicting how many winners will be from her class. Determine which probability distribution she should use given the following information.

(A) There are 386 tickets, one for each student. Tickets are placed back in the pool after being chosen and 5 tickets are drawn.

[BINOMIAL | HYPERGEOMETRIC]

(B) There are 386 tickets, one for each student. Tickets are removed from the pool after being chosen and 5 tickets are drawn.

[BINOMIAL | HYPERGEOMETRIC]

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EXAMPLE

A quality control manager wants to ensure that their staff's testing procedure will identify defects in shipments reliably, so they test the procedure on a shipment containing 100 units, 5 of which are defective, and have the staff complete the testing procedure of taking 20 random units in the shipment and identifying any defective units and replacing them in the shipment with working parts.

- (A) The company's retail partner will issue a complaint if a shipment contains 2 or more defects. What is the probability that the test identifies and removes enough defective products to avoid a complaint on the shipment?

New

$$P(x) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}}$$

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- (B) The quality control manager will adjust the testing procedure if the probability of removing enough defective units to avoid complaint is below 10%. Should they adjust their testing procedure?