Intro to Random Variables & Probability Distributions

- ◆ A Random Variable represents a single number determined by ______ for each outcome of an "experiment".
 - ► Discrete (DRV): #'s [CANNOT | CAN] be broken down further [e.g. Dice roll]
 - ► Continuous (CRV): #'s [CANNOT | CAN] be broken down further [e.g. height]
- ◆ A **Probability Distribution** shows the probabilities of _____ possible values that a random variable can be.

EXAMPLE

Verify that the table meets the criteria for a probability distribution.

Recall Freq. Distribu	ution	New Probability Distribution					
drank per Day f 0 10 1 20 2 40 3 20	eq. f 10 20 40 20	# Prizes won in random raffle X 0 1 2 3 4	Probability. P(X) 0.10 0.20 0.40 0.20 0.10	Criteria: 1)			

EXAMPLE You pay \$1 to play a random lottery. The profits and probabilities of each outcome are as follows.

(A) What is the missing probability in the table?

Lottery Profits				
Profit	Probability			
-\$1.00	0.40			
\$0.00	0.35			
\$5.00	?			
\$1,000,000.00	0.01			

(B) What is the probability of at least breaking even?

PRACTICE

Which of the following below is NOT a discrete random variable?

- (A) The number of customers who visit a coffee shop each day
- (B) The number of defective products in a shipment of 500 items
- (C) The number of employees in a company's human resources department
- (D) The annual revenue of a technology startup (in dollars)

EXAMPLE

In a random survey, people were asked how many sodas they drink per day.

(A) What is the probability a person responds with having at most 2 sodas per day?

Sodas per Day	Probability
0	0.50
1	0.31
2	0.09
3	0.05
4	0.03
5	0.01
6	0.01

(**B**) What is the probability a person responds with having **between 1 and 4** sodas per day?

Mean (Expected Value) of Random Variables

- ullet To find the mean μ or "**Expected Value**" E(X) of a DRV, ______ each value by its prob., then _____ results.
 - ▶ If not given a table, make one and include a column for $X \cdot P(X)$.

EXAMPLE

The table shows a probability distribution for the number of kids per household in a town. Find the expected value of this distribution.

# of kids (X)	0	1	2
Probability $P(X)$	0.15	0.60	0.25

Recall Mean of Data Set		Ne	New Mean of DRV with Probability Distribution			
Sodas per day 0 1 2	$\mu = \frac{\sum x}{N}$ $\frac{0+1+2}{3} = 1$		7 0 1 2 Exp. Va	P(X) 0.15 0.60 0.25 al. E(X):	$X \cdot P(X)$	$\mu = E(X) = \sum$

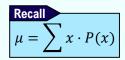
PRACTICE

A factory produces lightbulbs in batches of 50. The probability distribution for the number of defective lightbulbs in a randomly selected batch is shown below. Find the expected value.

# of Defective bulbs (X)	0	1	2	3	4	5
Probability $P(X)$	0.20	0.30	0.25	0.15	0.07	0.03

EXAMPLE

A stock trader buys a stock option. There is a 70% chance that the stock goes up, which would net a \$200 profit. If the stock goes down, they'll lose \$800. Find the expected value of the option's profit. Is this a good or bad trade?



Variance & Standard Deviation of Discrete Random Variables

- ◆ To find variance (σ^2) and standard deviation (σ) of a DRV, make a table with columns for $X \cdot P(X) \& X^2 \cdot P(X)$:
 - ▶ Recall: To find standard deviation, $\sigma = \sqrt{\sigma^2}$

New
$$\sigma^2 = \sum [X^2 \cdot P(X)] - \mu^2$$

$$= \sum [(X - \mu)^2 \cdot P(X)]$$
Easier to use!

EXAMPLE

The table shows a probability distribution for the number of kids per household in a town. Find the variance and standard deviation of this distribution.

# of kids (X)	0	1	2
Probability $P(X)$	0.15	0.60	0.25

Variance & Standard Deviation of DRV New Mean (μ) : $X^2 \cdot P(X)$ P(X) $X \cdot P(X)$ X μ^2 : 0.15 $0 \cdot 0.15 = 0$ $1 \cdot 0.60 = 0.60$ 1 0.60 $\sum X^2 \cdot P(X)$: 2 0.25 $2 \cdot 0.25 = 0.50$ Variance (σ^2) : $\mu = 1.10$ Std. Dev. (σ) :

PRACTICE

A company tracks the number of complaints they receive, where the random variable X is the number of complaints received daily. Find the variance & standard deviation of this distribution.

# of complaints (X)	0	1	2	3
Probability $P(X)$	0.45	0.30	0.20	0.05

Recall
$$\sigma^2 = \sum [X^2 \cdot P(X)] - \mu^2$$

EXAMPLE

When analyzing portfolios of financial assets, investment risk is typically calculated by finding the standard deviation of the probability distribution of potential outcomes (gains or losses) and their associated probabilities. Calculate the investment risk of the following company.

Loss Next Year (X)	\$0	\$500	\$1000	\$1500	\$2000
Probability $P(X)$	0.05	0.20	0.40	0.25	0.10

