## TOPIC: DISCRETE RANDOM VARIABLES
### Intro to Random Variables & Probability Distributions

◆ A **Random Variable** represents a single number determined by __________ for each outcome of an "experiment".

► Discrete (DRV): #'s **[ CANNOT | CAN ]** be broken down further [e.g. Dice roll]

► Continuous (CRV): #'s **[ CANNOT | CAN ]** be broken down further [e.g. height]

◆ A **Probability Distribution** shows the probabilities of ______ possible values that a random variable can be.

**EXAMPLE** Verify that the table meets the criteria for a probability distribution.

**Recall** — **Freq. Distribution**

| Cans of soda drank per Day | Freq. $f$ |
|---|---|
| 0 | 10 |
| 1 | 20 |
| 2 | 40 |
| 3 | 20 |
| 4 | 10 |

**New** — **Probability Distribution**

| # Prizes won in random raffle $X$ | Probability. $P(X)$ |
|---|---|
| 0 | 0.10 |
| 1 | 0.20 |
| 2 | 0.40 |
| 3 | 0.20 |
| 4 | 0.10 |

Criteria:

**1)** $\_\_\_ \le P(X) \le \_\_\_$ For any $X$

**2)** $\sum_{All\ X} P(X) = \_\_\_\_ = \_\_\_\_\%$

**EXAMPLE** You pay $1 to play a random lottery. The profits and probabilities of each outcome are as follows.

$(A)$ What is the missing probability in the table?

$(B)$ What is the probability of **at least** breaking even?

| Lottery Profits | |
|---|---|
| Profit | Probability |
| -$1.00 | 0.40 |
| $0.00 | 0.35 |
| $5.00 | ? |
| $1,000,000.00 | 0.01 |

## TOPIC: DISCRETE RANDOM VARIABLES

**PRACTICE** Which of the following below is NOT a discrete random variable?

**(A)** The number of customers who visit a coffee shop each day
**(B)** The number of defective products in a shipment of 500 items
**(C)** The number of employees in a company's human resources department
**(D)** The annual revenue of a technology startup (in dollars)

**EXAMPLE** In a random survey, people were asked how many sodas they drink per day.

$(\boldsymbol{A})$ What is the probability a person responds with having **at most** 2 sodas per day?

| Sodas per Day | Probability |
|---|---|
| 0 | 0.50 |
| 1 | 0.31 |
| 2 | 0.09 |
| 3 | 0.05 |
| 4 | 0.03 |
| 5 | 0.01 |
| 6 | 0.01 |

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$(\boldsymbol{B})$ What is the probability a person responds with having **between 1 and 4** sodas per day?

## TOPIC: DISCRETE RANDOM VARIABLES
### Mean (Expected Value) of Random Variables

- To find the mean $\mu$ or "**Expected Value**" $E(X)$ of a DRV, __________ each value by its prob., then _____ results.
  - If not given a table, make one and include a column for $X \cdot P(X)$.

**EXAMPLE** The table shows a probability distribution for the number of kids per household in a town. Find the expected value of this distribution.

| # of kids $(X)$ | **0** | **1** | **2** |
|---|---|---|---|
| Probability $P(X)$ | 0.15 | 0.60 | 0.25 |

**Recall** **Mean of Data Set**

| Sodas per day |
|---|
| 0 |
| 1 |
| 2 |

$$\mu = \frac{\sum x}{N}$$

$$\frac{0+1+2}{3} = 1$$

**New** **Mean of DRV with Probability Distribution**

| $X$ | $P(X)$ | $X \cdot P(X)$ |
|---|---|---|
| 0 | 0.15 | |
| 1 | 0.60 | |
| 2 | 0.25 | |
| Exp. Val. $E(X)$ : | | |

$$\mu = E(X) = \sum$$

**PRACTICE** A factory produces lightbulbs in batches of 50. The probability distribution for the number of defective lightbulbs in a randomly selected batch is shown below. Find the expected value.

| # of Defective bulbs $(X)$ | **0** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|---|
| Probability $P(X)$ | 0.20 | 0.30 | 0.25 | 0.15 | 0.07 | 0.03 |

## EXAMPLE

A stock trader buys a stock option. There is a 70% chance that the stock goes up, which would net a $200 profit. If the stock goes down, they'll lose $800. Find the expected value of the option's profit. Is this a good or bad trade?

**Recall**

$$\mu = \sum x \cdot P(x)$$

## TOPIC: DISCRETE RANDOM VARIABLES

### Variance & Standard Deviation of Discrete Random Variables

- To find variance $(\sigma^2)$ and standard deviation $(\sigma)$ of a DRV, make a table with columns for $X \cdot P(X)$ & $X^2 \cdot P(X)$:
  - Recall: To find standard deviation, $\sigma = \sqrt{\sigma^2}$

**New**

$$\sigma^2 = \sum[X^2 \cdot P(X)] - \mu^2$$

$$= \sum[(X-\mu)^2 \cdot P(X)]$$

← Easier to use!

**EXAMPLE**

The table shows a probability distribution for the number of kids per household in a town. Find the variance and standard deviation of this distribution.

| # of kids $(X)$ | 0 | 1 | 2 |
|---|---|---|---|
| Probability $P(X)$ | 0.15 | 0.60 | 0.25 |

**New** — **Variance & Standard Deviation of DRV**

| $X$ | $P(X)$ | $X \cdot P(X)$ | $X^2 \cdot P(X)$ |
|---|---|---|---|
| 0 | 0.15 | $0 \cdot 0.15 = 0$ | |
| 1 | 0.60 | $1 \cdot 0.60 = 0.60$ | |
| 2 | 0.25 | $2 \cdot 0.25 = 0.50$ | |
| | | $\mu = 1.10$ | |

Mean $(\mu)$: ____________

$\mu^2$: ____________

$\sum X^2 \cdot P(X)$: ____________

Variance $(\sigma^2)$: ____________

Std. Dev. $(\sigma)$ : ____________

**PRACTICE**

A company tracks the number of complaints they receive, where the random variable X is the number of complaints received daily. Find the variance & standard deviation of this distribution.

| # of complaints $(X)$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Probability $P(X)$ | 0.45 | 0.30 | 0.20 | 0.05 |

**Recall**

$$\sigma^2 = \sum[X^2 \cdot P(X)] - \mu^2$$

## TOPIC: DISCRETE RANDOM VARIABLES

### EXAMPLE

When analyzing portfolios of financial assets, investment risk is typically calculated by finding the standard deviation of the probability distribution of potential outcomes (gains or losses) and their associated probabilities. Calculate the investment risk of the following company.

| **Loss Next Year** $(X)$ | **\$0** | **\$500** | **\$1000** | **\$1500** | **\$2000** |
|---|---|---|---|---|---|
| **Probability** $P(X)$ | 0.05 | 0.20 | 0.40 | 0.25 | 0.10 |

**Recall**

$$\sigma^2 = \sum [X^2 \cdot P(X)] - \mu^2$$