TOPIC: SAMPLING DISTRIBUTION OF SAMPLE PROPORTION

Using the Normal Distribution to Approximate Binomial Probabilities

◆ Recall: In a binomial dist., n trials w/ 2 outcomes (S or F); x = # of successes, $p = 1 - q = P(S)$ constant.

► Instead, for $np \geq$ ___ & $nq \geq$ ____ use a z-score to find probabilities instead of the binomial formula.

EXAMPLE

The probability of someone voting for a particular candidate in a two-person election is 56%. Use a normal distribution to approximate the probability that more than 60 of a sample of 100 people vote for the candidate.

Recall **Prob. w/ Binomial Formula**

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$P(a < X < b) = \sum_{}^{b} P(X = x)$$

$$P(X > 60) = P(X = 60) + \cdots + P(X = 100)$$

$$= \binom{100}{60} 0.45^{60} 0.55^{40} + \cdots$$

$$+ \binom{100}{100} 0.45^{100} 0.55^{0}$$

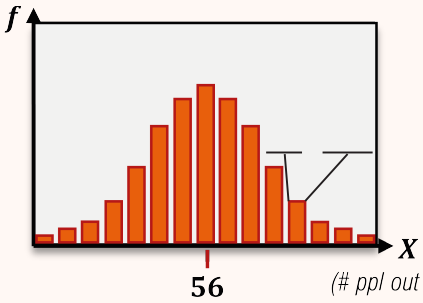
$$= ?$$

New **Prob. w/ Normal Approximation**

Recall

$$\mu_x = np,\ \sigma_x = \sqrt{npq}$$

$$z = \frac{x - \mu_x}{\sigma_x} = \frac{x -}{}$$



(# ppl out of 100 voting for candidate)

1) Verify $np \geq 5$ □
and $nq \geq 5$ □

2) Determine if you need to add or subtract 0.5 to x

3) Find z-score using equation above

4) Determine probability using a table or calculator

◆When approximating binomial distribution as normal, make a **continuity correction:**

Binomial	Normal
Exactly c	$P(c - 0.5 < x < c + 0.5)$
Between c & d	$P(c - 0.5 < x < d + 0.5)$
At most c	$P(x < c + 0.5)$
Fewer than c	$P(x < c - 0.5)$
At least c	$P(x > c - 0.5)$
More than c	$P(x > c + 0.5)$

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PRACTICE

A bank analyzes customer adoption of its new mobile banking app. Historically, 45% of customers use online banking services. Use a normal distribution to approximate the probability that between 62 and 70 customers out of a sample of 100 will adopt the online banking service.

PRACTICE

A previous study found that 80% of people preferred drinking Pepsi over Coca Cola. Use a normal distribution to approximate the probability that a random sample of 100 people reveals at least 60 people preferring Pepsi.

PRACTICE

A previous study found that 80% of people preferred drinking Pepsi over Coca Cola. Use a normal distribution to approximate the probability that, from this same random sample of 100 people, that between 10 and 11 people prefer Coca Cola.

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EXAMPLE

In 2023 about 35 million cars were recalled, out of approximately 94 million cars produced. A used car dealer has 76 cars that were produced in 2023. Use a normal distribution to approximate the probability that more than half of these cars were recalled.

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Sampling Distribution of Sample Proportion

◆ Recall: For binomial variables (X) when $np \geq 5$ & $nq \geq 5$, X has an approximately normal distribution.

► For a binomial experiment, the **Sample Proportion** $(\hat{p})$ is the actual # of successes out of the # of _____.

► Take many random samples to get a **sample distribution** of $\hat{p}$.

EXAMPLE You and several classmates take turns flipping a coin 10 times each. The distribution of heads (successes) is shown below. Find the mean and standard deviation of the sampling distribution for $\hat{p}$.

Recall **Distribution of Binomial Variable X**

$\mu_X = np, \sigma_X = \sqrt{npq}$

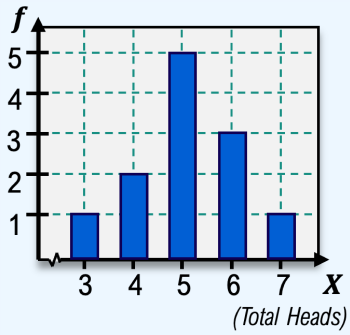
$n = 10, p = 0.5$

$\mu_X = 10 \cdot 0.5$

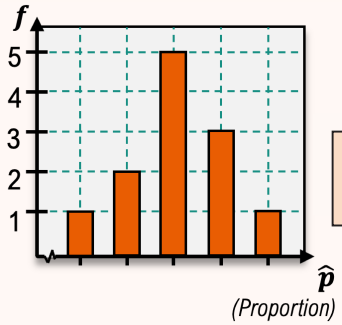
$= 5$

$\sigma_X = \sqrt{10 \cdot 0.5 \cdot 0.5}$

$= 1.58$



New **Distribution of Sample Proportions $\hat{p}$**



$\hat{p} =$ —

$\mu_{\hat{p}} =$ ___, $\sigma_{\hat{p}} = \sqrt{\quad\quad}$

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PRACTICE

A market research firm is studying customer satisfaction for a food delivery service. Based on past data, 85% of customers rate the service as "satisfactory". The firm randomly surveys groups of 250 customers. Find the mean $\mu_{\hat{p}}$ and standard deviation $\sigma_{\hat{p}}$ of the sampling distribution for $\hat{p}$. What would the shape of the sampling distribution be?