TOPIC: TWO MEANS – UNKNOWN, UNEQUAL VARIANCE

Difference in Means: Hypothesis Tests

◆ In hypothesis tests given ***TWO*** samples, we test claims about the ____________ between the means.

- ► Write H_0 as $\mu_1 =$ ______, i.e. $\mu_1 - \mu_2 =$____
- ► Find the P-value using the ____________ of $n_1 - 1$ & $n_2 - 1$ for the degrees of freedom.

EXAMPLE

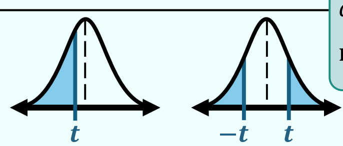
The table summarizes a study on the mean resting heart rate of males and females. Perform a hypothesis test using $\boldsymbol{\alpha = 0.05}$ to determine if there's a difference in between the two randomly sampled groups. Assume normal population distributions.

Mean Resting Heart Rate			
Group	**Sample Size (n)**	**Sample Mean ($\bar{x}$)**	**Sample Std Dev (s)**
Males	10	70.2 BPM	5.8 BPM
Females	11	81.4 BPM	6.4 BPM

Samples Random & Independent? ☐

σ_1 & σ_2 unknown & not equal? ☐

BOTH Samples Normal ***OR*** $n_1 > 30$, $n_2 > 30$ ☐

	1 Mean	2 Means
Hyp.	$H_0: \mu = \#$ $H_a: \mu$ </>/≠ #	$H_0: \mu_1 = \mu_2$ $H_a: \mu_1$ **[< | > | ≠]** μ_2
Test Stat. (σ unknown)	$t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$	$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$
P-Val.	$df = n - 1$ Area "beyond" t	 $df = \min\begin{Bmatrix} n_1 - 1 \\ n_2 - 1 \end{Bmatrix}$
Conclude	Because P-value **[< | >]** α, we **[REJECT | FAIL TO REJECT]** H_0. There is **[ENOUGH | NOT ENOUGH]** evidence to { restate H_a }	

1) H_0:

H_a:

2) **Males** **Females**

$n_1 =$ ___ , $n_2 =$ ___

$\bar{x}_1 =$ ___ , $\bar{x}_2 =$ ___

$s_1 =$ ___ , $s_2 =$ ___

$t =$

3) $df =$

P-value:

4) Because P-value **[< | >]** α, we **[REJECT | FAIL TO REJECT]** H_0.

There is **[ENOUGH | NOT ENOUGH]** evidence that there is a difference in the mean resting heart rate between males & females.

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PRACTICE

Researchers are comparing the average number of hours worked per week by employees at two different companies. Below are the results from two independent random samples. Assuming population standard deviations are unknown and unequal, calculate the t-score for the difference in means, but do not find a P-value or state a conclusion.

Company A: $n_1 = 25$; $\bar{x}_1 = 22.4$ hours; $s_1 = 3.2$ hours

Company B: $n_2 = 16$; $\bar{x}_2 = 21.1$ hours; $s_2 = 2.9$ hours

Recall

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

EXAMPLE

Use the information below to test the claim about the difference between two population means at the specified significance level. Assume samples are random and independent, normally distributed populations, and unknown and unequal population standard deviations.

Claim: $\mu_1 > \mu_2$; $\alpha = 0.10$

$n_1 = 32, \ \bar{x}_1 = 462, \ s_1 = 67$

$n_2 = 19, \ \bar{x}_2 = 431, \ s_2 = 85$

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Difference in Means: Confidence Intervals

- To make a Conf. Int. for $\mu_1 - \mu_2$, use point estimator $\bar{x}_1 - \bar{x}_2$ and margin of error:

New

$$E = t_{\alpha/2} \cdot \sqrt{\frac{\quad}{n_1} + \frac{\quad}{n_2}}$$

EXAMPLE

The table summarizes a study on the mean resting heart rate of randomly sampled males and females. Make a 90% confidence interval for the difference in mean heart rate between males and females. Assume normal population distributions. What does this result suggest about the claim that there is no difference in mean resting heart rate between males & females?

Mean Resting Heart Rate			
Group	**Sample Size (n)**	**Sample Mean ($\bar{x}$)**	**Sample Std Dev (s)**
Males	10	70.2 BPM	5.8 BPM
Females	11	81.4 BPM	6.4 BPM

We are ______ % confident that the true difference in mean resting heart rate in males and females is between ________ & ________. Because this **[DOES | DOES NOT]** include 0, we **[REJECT | FAIL TO REJECT]** $H_0: \mu_1 = \mu_2$. There is **[ENOUGH | NOT ENOUGH]** evidence that there is a difference...

HOW TO: Make a Confidence Interval for $p_1 - p_2$

1) Verify samples are independent **AND** random ☐

Normal ***OR*** $n_1 > 30$, $n_2 > 30$ ☐

2) Find critical value: $t_{\alpha/2}$

3) Point estimate: $\bar{x}_1 - \bar{x}_2$

4) Margin of Error: E

5) Find upper & lower bounds

$\left((\bar{x}_1 - \bar{x}_2) - E, (\bar{x}_1 - \bar{x}_2) + E\right)$

Recall

$$df = \min\begin{Bmatrix} n_1 - 1 \\ n_2 - 1 \end{Bmatrix}$$

- If Conf. Int. *DOESN'T* include 0, we're confident of a *DIFFERENCE* between μ_1 & μ_2, so we ____________ H_0.
- If Conf. Int. *DOES* include 0, it's possible there is *NO DIFFERENCE* between μ_1 & μ_2, so we ____________ H_0.

TOPIC: TWO MEANS – UNKNOWN, UNEQUAL VARIANCE

PRACTICE

A researcher is comparing average number of hours slept per night by college students who work part-time versus those who don't. From survey data, they calculate $\bar{x}_1 = 6.82$ hours and $\bar{x}_2 = 6.57$ hours with a margin of error of 0.41. Should they reject or fail to reject the claim that there is no difference in hours slept between the two groups?

(A) Reject

(B) Fail to reject

(C) There is not enough information to answer the question