TOPIC: BINOMIAL DISTRIBUTION

The Binomial Experiment

◆ A common example of a Disc. Rand. Var. (X) is the outcomes of a binomial experiment:

   ▶ "Binomial" = events with ___ outcomes: success & failure (e.g. heads/tails in coin flip). x = # of _____

   ▶ "Experiment" = Series of n independent _____; each trial has P(success)= __ , P(failure) $= q =$ _____

> **Recall**
> $$P(A') = 1 - P(A)$$

EXAMPLE	Determine whether the following experiments are binomial experiments. If they are, identify the values of $n, p, q,$ and x.

(A) You flip a coin 4 times. You count the number of times the coin lands on heads, which is 3 times.

   ☐ Only 2 outcomes?

   ☐ Fixed # of trials?

   ☐ Independent trials?

   ☐ Equal P(success) per trial?

(B) You pull 4 marbles out of a bag of red & blue marbles *without* replacing them. The chance of getting a blue marble is 60%. You get one 1 blue marble.

   ☐ Only 2 outcomes?

   ☐ Fixed # of trials?

   ☐ Independent trials?

   ☐ Equal P(success) per trial?

PRACTICE	A basketball player normally has a 70% chance of making a free throw. The player shoots until finally making a basket, where x is the number of shots they take. Is this a binomial experiment?

(A) Yes
(B) No, there are not 2 outcomes
(C) No, there are not a fixed # of trials
(D) No, the trials are not independent
(E) No, the probability of success is not the same for each trial

TOPIC: BINOMIAL DISTRIBUTION

You roll a six-sided die 10 times and record the number landed on for each roll as the random variable. Is this a binomial experiment?

(A) Yes

(B) No, there are not 2 outcomes

(C) No, there are not a fixed # of trials

(D) No, the trials are not independent

(E) No, the probability of success is not the same for each trial

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Using the above example, now you roll another 10 times and choose the random variable as the number of times you roll exactly a 2. Is this a binomial experiment?

(A) Yes

(B) No, there are not 2 outcomes

(C) No, there are not a fixed # of trials

(D) No, the trials are not independent

(E) No, the probability of success is not the same for each trial

EXAMPLE

A pharmaceutical company claims the probability of gaining immunity from a new vaccine is 80%. The vaccine was given to 100 patients, and 83 gained immunity. Is this a binomial experiment? If so, identify values of $n, p, q,$ and x.

TOPIC: BINOMIAL DISTRIBUTION
Probabilities of Exact Successes in a Binomial Distribution

◆ You will often have to calculate the **probability** of getting X successes $P(X = x)$ in a binomial experiment.

EXAMPLE

You put red & blue marbles in a bag. You take out one marble randomly and *replace* it, repeating this for a total of 4 times. If the probability of getting a red marble is 80%, what is the probability of getting exactly 3 red marbles?

New — **Probability Distribution**

1 Trial = _____   Success = _____   Failure = _____

$n =$ _____ $p =$ _____ $q =$ _____

$x =$ _____ :

$P(X = 3) = P(\_) \cdot P(\_) \cdot P(\_) \cdot P(\_) \cdot (\text{# ways to } \_\_\_\_\_ \text{ 3 R's})$

$$P(X = x) = P(x) = \left(\quad\right) \cdot \_\_^{\_} \cdot \_\_^{\_\_\_}$$

(Binomial Probability Formula)

HOW TO: Find Prob. for Var with Binomial Distribution

1) Define *trial*, *success*, & *failure*
2) Identify $n, p, q, \& x$
   $n =$ # trials
   $p = P(success)$
   $x =$ "desired" # of successes
3) Use formula to evaluate
   $$P(X = x)$$

Probability of multiple events:

Recall
$$P(A \cap B) = P(A) \cdot P(B)$$
*A and B are independent

(Multiplication Rule)

ways to arrange k out of n objects:

Recall
$$_nC_k = \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

(Combination)

PRACTICE

You take a 6-question quiz with True/False questions. What is the probability of getting all 6 questions correct by simply guessing?

Recall

$$P(X = x) = \binom{n}{x} \cdot p^x \cdot q^{n-x}$$

(Binomial Probability)

Recall

$${}_nC_k = \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

(Combination)

PRACTICE

National surveys indicate that 36% of people have been in a car accident in the last 5 years. If you randomly sample 10 people, how likely is it that exactly 4 have had an accident in the last 5 years?

TOPIC: BINOMIAL DISTRIBUTION

Mean and Standard Deviation of Binomial Distribution

◆ Instead of using summations, find μ_x & σ_x of any binomial probability distribution using just n & P.

A company offers a new product, and 60% of customers are expected to purchase it after a demonstration. If you randomly select 10 customers, how many would you expect to purchase the product? What is the standard deviation and variance of this distribution?

Recall	Exp. Value & Std. Dev. for any DRV	New	Exp. Value & Std. Dev. of Binomial Dist.

$$\mu = E(X) = \sum x_i \cdot P(x_i)$$

$$\sigma = \sqrt{\sum [X^2 \cdot P(X)] - \mu^2}$$

$\mu = 1 \cdot P(X = 1) + 2 \cdot P(X = 2) +$

$\qquad \dots + 6 \cdot P(X = 6)$

$\mu_x =$ _____

$\sigma_x =$ _____

$n =$ ____

$p =$ ____

PRACTICE

Based on historical weather data in a certain city, about 62% of the days are cloudy. Find the mean, standard deviation, and variance for the number of cloudy days in a 30-day month.

Recall

$\mu_x = np$

$\sigma_x = \sqrt{npq}$

Multiple Probabilities in Binomial Distributions

◆ When you see keywords such as "**between**" or "**at least**", simply _____ probabilities.

Common Terms for Finding Probabilities					
"exactly a"	"less than b"	"at most b"	"between a & b"	"at least a"	"more than a"
$P(X = a)$	$P(0) +$ $... P(X = b - 1)$	$P(0) +$ $... P(X = b)$	$P(X = a) +$ $... P(X = b)$	$P(X = a) +$ $... P(X = n)$	$P(X = a + 1) +$ $... P(X = n)$

EXAMPLE

A marketing company wants to analyze how many customers will open a promotional email, so they send it to 10 customers. Based on past campaigns, 40% of people typically open the email.

(A) What is the probability that between 0-2 customers open the email?

Recall

$$P(X = x) = \binom{n}{x} \cdot p^x \cdot q^{n-x}$$

(Binomial Distribution)

(B) What is the probability that at least 3 open the email?

Recall

$$P(A') = 1 - P(A)$$

(Prob. of Complement)

◆ Sometimes it may be easier to *subtract* the probability of the complement.

A gardener plants 8 seeds, and each as a 65% probability of germinating successfully. Find the probability that less than 4 seeds germinate successfully.

Recall

$$P(X = x) = \binom{n}{x} \cdot p^x \cdot q^{n-x}$$

(Binomial Distribution)

TOPIC: BINOMIAL DISTRIBUTION

Finding Binomial Probabilities Using a TI-84

◆ For *exact* probabilities, use `binompdf`. For *cumulative* prob's (<, >, "at least", "between", etc.) use `binomcdf`.

▶ The `binomcdf` gives the probability of values _____ `x value`.

EXAMPLE

A manufacturing company produces light bulbs, with a defect rate of 5%. The company produces 1,000 light bulbs in a day. Find each probability.

$n =$ _____ , $p =$ _____

(A) Exactly 50 light bulbs are defective.

_____

(B) At most 50 light bulbs are defective.

_____

(C) Less than 50 light bulbs are defective.

HOW TO: Find Binomial Probabilities on TI-84

1) 2ND VARS DISTR

2) V If Exact: `A:binompdf(`

   If Cumulative: `B:binomcdf(`

3) `trials:`

   `p:`

   `x value:`

PRACTICE

A biologist is monitoring a large bird sanctuary where a particular bird species is known to have a 70% success rate for each nesting attempt (at least one chick fledges from the nest). This season, she observes 500 independent nesting attempts across the sanctuary.

(A) What is the probability that exactly 450 nesting attempts are successful?

_____

(B) What is the probability that less than 330 attempts are successful?

_____

(C) What is the probability that at least 330 nesting attempts are successful?

_____

(D) What is the probability that 330 – 370 nesting attempts are successful?

HOW TO: Find Binomial Probabilities on TI-84

1) 2ND VARS DISTR

2) V If Exact: `A:binompdf(`

   If Cumulative: `B:binomcdf(`

3) `trials:`

   `p:`

   `x value:`

TOPIC: BINOMIAL DISTRIBUTION

A retail company sends out promotional emails and finds that approximately 1.5% of emails sent through their system are flagged as spam by users. On a given day, 20,000 emails are sent through the system.

(A) What is the probability that more than 350 emails are flagged as spam?

(B) If more than 350 emails are flagged as spam, should this be investigated?

HOW TO: Find Binomial Probabilities on TI-84

1) [2ND] [VARS] DISTR

2) [v] If Exact: **A:binompdf(**

   If Cumulative: **B:binomcdf(**

3) **trials:**

   p:

   x value:

Recall
$$P(A') = 1 - P(A)$$

EXAMPLE

A company tracks the quality of its customer service calls. Based on historical data, the company estimates that 90% of all customer service calls result in a satisfied customer. On a randomly selected day, 20 customer service calls are made.

(A) What is the probability of 15 or fewer customers out of the 20 being satisfied?

Recall
$$P(X = x) = (n, k) \cdot p^x \cdot q^{n-x}$$
$$\mu_x = np, \qquad \sigma_x = \sqrt{npq}$$
(Binomial Distribution)

(B) Use the range rule of thumb to determine whether observing 15 or fewer satisfied customers is statistically unusual. Should the manager be concerned about a possible decline in service quality?

SIGNIFICANT   SIGNIFICANT

μ

$\mu - 2\sigma$ $\mu + 2\sigma$

⊢—NOT SIG—⊣
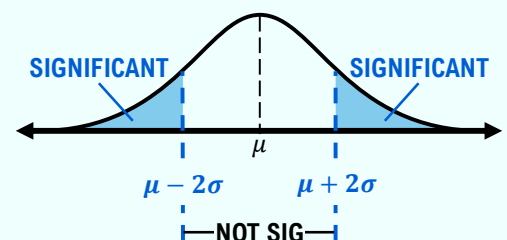
TOPIC: BINOMIAL DISTRIBUTION

A medical researcher is studying the effectiveness of a new vaccine. In clinical trials, the vaccine is known to be 90% effective in preventing infection when administered properly. The researcher selects a random sample of 10 patients who received the vaccine and checks whether they became infected after exposure.

(A) Find the probability that the vaccine was effective for 7 or fewer patients.

Recall

$$P(X = x) = \binom{n}{x} \cdot p^x \cdot q^{n-x}$$

(Binomial Distribution)

Recall

$$P(A') = 1 - P(A)$$

(B) If the researcher finds the vaccine was effective for 7 or fewer patients, is this result "statistically significant"?
Use levels of significance: 10%, 5%, & 1%. *(Hint: compare prob. to each level of significance)*

$P(X \leq 7) =$ _____ [< | >] 1%,   so [SIG. | NOT SIG.]

$P(X \leq 7) =$ _____ [< | >] 5%,   so [SIG. | NOT SIG.]

$P(X \leq 7) =$ _____ [< | >] 10%,   so [SIG. | NOT SIG.]