TOPIC: POISSON DISTRIBUTION

Intro to Poisson Distribution

◆ Recall: **Binomial** Variable: $X = \#$ of successes out of n trials, with $P(\text{success}) = p$.

► **Poisson** Variable: $X = \#$ of occurences in a given __________ (usually time), with mean *rate* of occurence $= \lambda$.

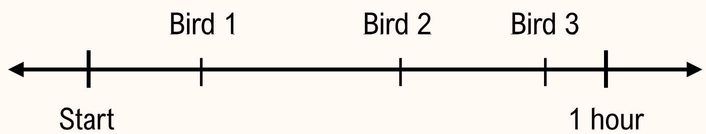
EXAMPLE

A student studying bird behavior observes a feeder for **1 hour** and knows from past data that the average rate of birds landing on the feeder is **3.6 birds per hour**. Determine what distribution would be used to represent the # of birds that land on the feeder within the hour.

Recall — Binomial Experiment

- ☐ Only 2 outcomes?
- ☐ Fixed # of trials?
- ☐ Independent trials?
- ☐ Equal $P(\text{success})$ per trial?

New — Poisson Experiment



- ☐ Fixed time interval? — Time Interval =
- ☐ Independent int's? — 1 Occurrence =
- ☐ Equal $P(\text{occurrence})$ @ any time? — λ = mean # of events in interval = _______

PRACTICE

A baker wants to predict how many customers will enter their bakery. Determine which probability distribution they should use given the following information.

(***A***) There is a 10% chance that any one person who walks by will enter the bakery and 20 people walk by.

[BINOMIAL | POISSON]

(***B***) On average, 2 customers come into the bakery every 15 minutes.

[BINOMIAL | POISSON]

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Finding Probabilities Using the Poisson Distribution

◆ You'll often have to calculate the probability of X occurrences $P(X)$ in a Poisson experiment with mean rate λ.

EXAMPLE

A student observes the feeder for 1 hour and knows from past data that the average rate of birds landing on the feeder is 3.6 birds per hour. Find the probability that exactly 3 birds land on the feeder and the mean & standard deviation of $X =$ # of birds landing on the feeder.

New **Poisson Distribution**

☐ Fixed time interval? Time Interval =

☐ Independent int's? 1 Occurrence =

☐ Equal P(occurrence) @ any time? λ = mean # of events in interval = ______

$$P(x) = \frac{(\lambda^x \cdot e^{-\lambda})}{x!}$$

$\mu =$ ___, $\sigma^2 =$ ____

PRACTICE

A small electronics retailer tracks the number of customers who arrive to purchase replacement phone chargers. Based on historical data, the store finds that, on average, 3 customers per day buy a charger. The store manager wants to use this information to optimize inventory decisions and reduce the risk of stockouts.

$(\boldsymbol{A})$ Find the probability that 5 customers buy a charger in a given day.

Recall

$$P(x) = \frac{(\lambda^x \cdot e^{-\lambda})}{x!}$$

(Poisson Dist.)

$(\boldsymbol{B})$ If the store stocks 5 chargers per day, find the probability that they will have inventory remaining on a given day.

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PRACTICE

A quality control inspector at a textile factory is examining long rolls of fabric for defects. The inspector knows from past experience that, on average, there are 0.5 defects per meter of fabric. What is the probability that the inspector finds 0 defects in any given meter of fabric?

EXAMPLE

A small electronics retailer tracks the number of customers who arrive to purchase replacement phone chargers. Based on historical data, the store finds that, on average, 3 customers per day buy a charger. The store manager wants to use this information to optimize inventory decisions and reduce the risk of stockouts.

$(\boldsymbol{A})$ If the store stocks 4 chargers per day, what is the probability that demand exceeds supply on any given day?

Recall

$$P(x) = \frac{(\lambda^x \cdot e^{-\lambda})}{x!}$$

(Poisson Dist.)

$(\boldsymbol{B})$ The store is considering a bulk purchase discount from a supplier but only wants to order extra inventory if the probability of needing more than 6 chargers in a day is greater than 10%. Should they order the extra inventory?

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EXAMPLE

A customer service call center receives an average of 12 calls per hour. The management wants to understand call volume patterns to make better staffing decisions. One employee can typically handle four calls per 30 minutes.

$(\boldsymbol{A})$ How many calls should the management expect in a given 30 min period?

Recall

$$P(x) = \frac{(\lambda^x \cdot e^{-\lambda})}{x!}$$

$$\mu = \lambda, \qquad \sigma^2 = \lambda$$

(Poisson Dist.)

$(\boldsymbol{B})$ If only 1 person is staffed for the last 30 min of the day, what is the probability that there will be more than 4 calls?

$(\boldsymbol{C})$ From $(\boldsymbol{B})$, how many standard deviations from the mean is 4 calls in a half hour? Should management have more than 1 staff member available in any given 30 min period?

TOPIC: POISSON DISTRIBUTION

Using the Poisson Distribution to Approximate Binomial Probabilities

- Estimate Binomial probabilities with large numbers using the Poisson distribution. Substitute λ with

Recall
$\mu = np$
(Binom. Mean)

  - Requirements: $n \geq$ ______ AND $np \leq$ ______

EXAMPLE

In a school raffle, each ticket has a 1/500 chance of winning a prize. At the school event, 600 students each buy one ticket. Use a Poisson dist. to estimate the prob. that 2 students win prizes.

☐ $n \geq 100$?

☐ $np \leq 10$?

$n =$ ______

$p =$ ______

$\lambda =$ ______

Recall
$$P(x) = \frac{(\lambda^x \cdot e^{-\lambda})}{x!}$$
(Poisson Dist.)

PRACTICE

A financial analyst is assessing the risk of credit defaults in a large bond portfolio. The portfolio contains 2,000 corporate bonds, & the probability of any one bond defaulting in a year is 0.002.

(A) Can the # of bonds which will default be approximated using the Poisson distribution? If so, find λ.

Recall
$\mu = np$
(Binom. Mean)

(B) Use the Poisson distribution to estimate the probability that more than 5 bonds default in a year.

Recall
$$P(x) = \frac{(\lambda^x \cdot e^{-\lambda})}{x!}$$
(Poisson Dist.)

(C) The analyst considers any probabilities less than 5% to be significant. If more than 5 bonds default in a year, should the analyst be concerned?

TOPIC: POISSON DISTRIBUTION

Finding Poisson Probabilities Using a TI-84

- For *exact* probabilities, use **`poissonpdf`**. For *cumulative* prob's (<, >, "at least", etc.), use **`poissoncdf`**.
  - The **`poissoncdf`** gives the probability of values ____ **`x value`**.

EXAMPLE

A student working on a transportation engineering project analyzes traffic flow at an intersection. From past data, the average # of cars per minute is 17.6. Find each probability.

(***A***) Exactly 15 cars go through the intersection during the first minute.

____

(***B***) At most 15 cars go through the intersection during the first minute.

HOW TO: Find Poisson Probabilities Using TI-84

1) 2ND VARS **DISTR**

2) V If Exact: **`D:poissonpdf(`**

If Cumulative: **`E:poissoncdf(`**

3) enter parameter & x-val

`λ:`

`x value:`

PRACTICE

A student working on a transportation engineering project analyzes traffic flow at an intersection for 20 min. From past data, the average # of cars per minute is 17.6.

(***A***) What is the expected number of cars in the entire 20 min period?

____

(***B***) Find the probability that the student observes 350 or more cars total.

HOW TO: Find Poisson Probabilities Using TI-84

1) 2ND VARS **DISTR**

2) V If Exact: **`D:poissonpdf(`**

If Cumulative: **`E:poissoncdf(`**

3) enter parameter & x-val

`λ:`

`x value:`