TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

Population Standard Deviation (σ) Known

◆ To make a conf. int. for μ when σ is known, use $\bar{x}$ as a point estimate & margin of error:

New

$$E = z_{\alpha/2} \cdot \text{——}$$

► Recall: $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ has a standard normal distribution when X is normal ***OR*** sample size $n > 30$

EXAMPLE

Over 36 trips to work, you find a sample mean of 1 hour. Construct a 90% confidence interval for the true (population) mean travel time.

Use a population standard deviation of 18 minutes.

We are ___ % confident that the mean time to travel to work is between _____ & _____.

HOW TO: Make a Confidence Interval for μ (σ Known)

1) Verify: Sample is random ☐

X is normal ***OR*** $n > 30$ ☐

2) Find critical value: $z_{\alpha/2}$

3) Margin of Error: $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

4) Find upper & lower bounds

$$(\bar{x} - E, \bar{x} + E)$$

EXAMPLE

You consider how much time it takes to find a parking spot and find a mean of 1 hour and 22 minutes, with a population standard deviation of 15 minutes. Assume this information was based on a sample of 100 different days of circling the parking garage. Construct and interpret a 99% confidence interval.

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PRACTICE

A retailer wants to estimate the average amount spent by customers on holiday shopping. In a random sample of 50 customers, the average amount spent was \$250, and the population standard deviation σ is known to be \$40. Construct an interpret an 80% confidence interval for the average amount spent by all customers.

HOW TO: Make a Confidence Interval for μ (σ Known)

1) Verify: Sample is random ☐

X is normal ***OR*** $n > 30$ ☐

2) Find critical value: $z_{\alpha/2}$

3) Margin of Error: $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

4) Find upper & lower bounds

$(\bar{x} - E, \bar{x} + E)$

PRACTICE

Books get more and more expensive every semester, but the distribution of their prices is always normal. 25 randomly selected students in your school spent a mean amount of \$500 with a population standard deviation of \$50. Construct and interpret a 98% confidence interval for the true mean spending on books.

PRACTICE

You want to purchase one of the new Altimas. You randomly select 400 dealerships across the United States and find a mean of \$25,000. Assume a population standard deviation of \$2500. Construct and interpret a 94% confidence interval for the true mean price for the new Nissan Altima.

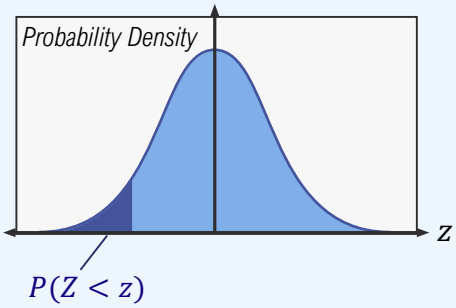
TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

Critical Values: t-Distribution

◆ To create a conf. int. for μ when σ is unknown, use s to estimate σ. In this case, $\dfrac{\bar{X}-\mu}{/\sqrt{n}}$ follows a ***t*-distribution**.

► Finding a critical value $t_{\alpha/2}$ also requires **Degrees of Freedom**. Look up in a *t*-table or calculator.

Recall — Normal Distribution



$\mu_z = 0, \sigma_z = 1$

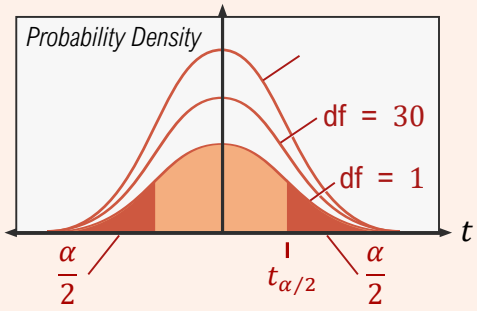
$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$$

has a standard normal dist.

z-table: Use _____ to find ***z***

z	**.00**	**...**	**.05**	**.06**	**.07**	**...**
...	...	...	...	...	...	...
-2.0	.0228	...	.0202	.0197	.0192	...
-1.9	.0287	...	.0256	.0250	.0244	...
...	...	...	...	...	...	...

New — *t*-Distribution



$\mu_t = 0, \sigma_t$ 1

$$\frac{\bar{X}-\mu}{s/\sqrt{n}}$$

has t-distribution with _____ d.f.

t-table: Use _____ & _____ to find ***t***.

Conf. Lev.	**50%**	**...**	**90%**	**95%**	**99%**
↓ df | $\alpha/2 \rightarrow$	**0.50**	**...**	**0.05**	**0.025**	**0.005**
1	1.000	...	6.314	12.71	63.66
2	0.816	...	2.920	4.303	9.925
...	...	...	...	...	...
30	0.683	...	1.697	2.042	2.750

EXAMPLE Find & compare the critical values for $(\boldsymbol{A})$ a 95% confidence interval with $\boldsymbol{\alpha/2} = 0.025$ where σ is known & $(\boldsymbol{B})$ where σ is unknown, using a sample size of 31.

$(\boldsymbol{A})$

$(\boldsymbol{B})$

◆ You can only use $z_{\alpha/2}$ or $t_{\alpha/2}$ in a confidence interval if the sample is random & X is normal OR sample size $n > 30$.

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PRACTICE Answer the questions below.

(***A***) Find the critical value $t_{\alpha/2}$ for an 80% confidence interval given a sample size of 51.

(***B***) Find the critical value $t_{\alpha/2}$ for an 95% confidence interval given a sample size of 6.

EXAMPLE For each problem below determine if you can construct a confidence interval for μ using the standard normal or t-distribution. If so, which should be used? If not, why not?

(***A***) You randomly select 18 test scores from a Statistics class. The sample mean of the test scores is 74 and the population standard deviation is 6. Construct a 99% confidence interval for the population mean of the test scores.

(***B***) You randomly measure the height of 87 men and find an average height of 5'10" with a population standard deviation of 5 inches. Assuming the heights are normally distributed, construct a 99% confidence interval for the population mean height.

(***C***) 25 randomly selected students in your school spent on average $500 on textbooks with a sample standard deviation of $50. Assuming the spending on books is normally distributed, construct a 98% confidence interval for the population mean textbook spending.

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PRACTICE

For which of the following scenarios can you NOT create a confidence interval using the standard normal or t-distribution?

($\boldsymbol{A}$) A real estate agent wants to estimate the average selling price of homes in a neighborhood. She collects random sample data from 8 recent home sales and finds that the sample mean selling price is $350,000 with a sample standard deviation of $25,000. Assuming home prices in the neighborhood are normally distributed, construct at 95% confidence interval for the population mean selling price.

($\boldsymbol{B}$) A financial analyst is studying the average return on investment (ROI) for a particular stock. He randomly selects data from 30 investors and finds that the sample mean ROI is 6.2% with a population standard deviation of 1.5%. Construct a 90% confidence interval for the population mean ROI.

($\boldsymbol{C}$) An economist is analyzing monthly utility expenses for households. He collects data from 12 households and calculates a sample mean monthly expense of $180 with a sample standard deviation of $15. He also notes that the distribution of utility expenses is heavily skewed to the right. Construct a 90% confidence interval for the population mean monthly utility expense.

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

Using TI-84 for the t-Distribution

◆ To find a t-value ($t_{\alpha/2}$) from significance level (α), use the **`4:invT`** function.

EXAMPLE Find the critical val. ($t_{\alpha/2}$) for an 80% conf. int. given $n = 51$.

$C =$ _____ $\alpha =$ _____ $\alpha/2 =$ _____

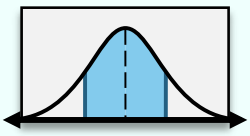
$df =$ _____

$-t_{\alpha/2} =$ _____ $t_{\alpha/2} =$ _____

Recall

$\alpha = 1 - C$

$df = n - 1$



HOW TO: Find t-Value From Prob. on TI-84

1) 2ND VARS (Distr), **`4:invT`**

2) Enter **`area:`** *(to left)*

`df:`

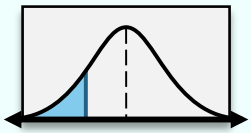
3) **`paste`**, ENTER

◆ To find a probability from t-value, use the **`6:tcdf`** function.

EXAMPLE Find $P(T < -1.47)$ given $df = 50$.

lower = _____

upper = _____



HOW TO: Find Prob. From t-Value on TI-84

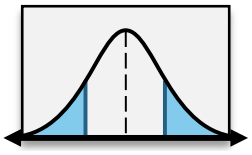
1) 2ND VARS (Distr), **`6:tcdf`**

2) Enter **`lower:`** (**`-1E99`** for -∞)

`upper:` (**`1E99`** for ∞)

`df:`

3) **`paste`**, ENTER



PRACTICE Fill out the table using a calculator and $n = 30$.

Conf Level (C)	Sig. Level (α)	Area to Left $\left(\frac{\alpha}{2}\right)$	$df = n - 1$	Critical Val. ($t_C = t_{\alpha/2}$)
90%	0.10			
95%	0.05			
99%	0.01			

HOW TO: Find t-Value From Prob. on TI-84

1) 2ND VARS (Distr), **`4:invT`**

2) Enter **`area:`** *(to left)*

`df:`

3) **`paste`**, ENTER

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

PRACTICE Find the critical val. ($t_{\alpha/2}$) for each confidence interval.

($\boldsymbol{A}$) Confidence Level $= 95\%, n = 35$

($\boldsymbol{B}$) Confidence Level $= 90\%, n = 48$

HOW TO: Find t-Value From Prob. on TI-84

1) 2ND VARS (Distr), `4:invT`

2) Enter `area:` *(to left)*

`df:`

3) `paste`, ENTER

PRACTICE Find each probability.

($\boldsymbol{A}$) $P(T > 1.5), df = 8$

($\boldsymbol{B}$) $P(-1.5 < T < 1.5), df = 8$

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

Population Standard Deviation (σ) Unknown

◆ To create a confidence interval for μ when σ is unknown, use s & $t_{\alpha/2}$ instead of σ & $z_{\alpha/2}$ to find margin of error.

► Recall: $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ has a t-distribution with df = $n-1$, ONLY when X is normal ***OR*** sample size $n > 30$.

EXAMPLE

Over 36 trips to work, you find a sample mean of 1 hour & a sample standard deviation of 21 minutes. Construct & interpret a 90% confidence interval for the true (population) mean travel time.

We are ___ % confident that the mean time to travel to work is between _____ & _____.

HOW TO: Make a Confidence Interval for μ (σ Known)

1) Verify: Sample is random ☐

X is normal ***OR*** $n > 30$ ☐

2) Critical Val. (use table/calculator):

σ known: $z_{\alpha/2}$ ***OR*** *unknown*: _____

3) Margin of Error:

$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ ***OR*** $E = t_{\alpha/2} \cdot \frac{}{\sqrt{n}}$

4) Find upper & lower bounds

$(\bar{x} - E, \bar{x} + E)$

PRACTICE

You ask 16 people in your Statistics class what their grade is. The data appears to be distributed normally. You find a sample mean and sample standard deviation of 60 and 24, respectively. Construct and interpret a 95% confidence interval for the population mean class grade.

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

EXAMPLE

An HR manager wants to estimate the average number of sick days taken by employees per year. They select a random sample of employees and find the sample mean is 6.8 days, with a standard deviation of 2.4 days.

(***A***) Construct a 90% confidence interval for the mean sick days taken by all employees in the company if the sample was $n = 15$ employees.

HOW TO: Make a Confidence Interval for μ (σ Known)

1) Verify: Sample is random ☐

X is normal ***OR*** $n > 30$ ☐

2) Critical Val. (use table/calculator):

σ known: $z_{\alpha/2}$ ***OR*** *unknown*: _____

3) Margin of Error:

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \quad \textbf{OR} \quad E = t_{\alpha/2} \cdot \frac{}{\sqrt{n}}$$

4) Find upper & lower bounds

$$(\bar{x} - E, \bar{x} + E)$$

(***B***) Now suppose the sample was $n = 41$ employees. Construct a 90% confidence interval.

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PRACTICE

You want to purchase one of the new Altimas. You randomly select 400 dealerships across the United States and find a mean of $25,000 and sample standard deviation of $2500. Construct and interpret a 94% confidence interval for the true mean price for the new Nissan Altima.

HOW TO: Make a Confidence Interval for μ (σ Known)

1) Verify: Sample is random ☐

X is normal ***OR*** $n > 30$ ☐

2) Critical Val. (use table/calculator):

σ known: $z_{\alpha/2}$ ***OR*** *unknown*: $t_{\alpha/2}$

3) Margin of Error:

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \quad \textbf{OR} \quad E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

4) Find upper & lower bounds

$$(\bar{x} - E, \bar{x} + E)$$

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

C.I. Using TI-84 – Mean

◆ To make a C.I. for the population mean using a calculator, use the **7:ZInterval** or **8:TInterval** function.

EXAMPLE Make a 95% C.I. for the mean in each data set.

(***A***)

$$\bar{x} = 23.67,\ \sigma = 5.724,\ n = 100$$

(***B***)

x-values							
21.7	18.42	25.3	13.85	19.1	22.68	16.7	24.59

HOW TO: Make a C.I. for Mean on TI-84

1) If given data, enter in **L1**
(STAT, **1:EDIT…**)

2) STAT > **TESTS**

3) If σ Known: **7: ZInterval**
If σ Unknown: **8: TInterval**

4)

If given data:	If given $\bar{x}, n, \ldots$:
DATA	**STATS**
σ: *(If Known)*	**σ:** *(If Known)* *or* **Sx:** *(If Unknown)*
List: L1	$\bar{x}$**:**
Freq: 1	**n:**
C-Level:	**C-Level:**

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

PRACTICE

A nutritionist wants to estimate the average grams of protein in a brand of protein bars. She takes a random sample of 40 protein bars with $\bar{x} = 210$ g & knows from prior data that $\sigma = 12$. Make a 95% conf. int. for μ.

We are _____% confident the protein bars have between (_____ , _____) calories on average.

HOW TO: Make a C.I. for Mean on TI-84

1) If given data, enter in **L1**
(STAT, **1:EDIT…**)

2) STAT > **TESTS**

3) If σ Known: **7: ZInterval**
If σ Unknown: **8: TInterval**

4)

If given data:	If given $\bar{x}, n, \ldots$:
DATA	**STATS**
σ: *(If Known)*	**σ:** *(If Known)* *or* **Sx:** *(If Unknown)*
List: L1	$\bar{x}$**:**
Freq: 1	**n:**
C-Level:	**C-Level:**

PRACTICE

A company wants to eliminate the average time employees spend on a training module. 15 employees are randomly selected, & their completion times recorded. The sample of times to complete the module is normally distributed with a mean 42.7 min & std. dev. 5.4 min.

(***A***) Make a 95% conf. int. for the true mean time to complete the training.

(***B***) Should management be concerned that the training might take too long if their policy states the module should take at most 45 min? Justify your answer using your confidence interval.

We are _____% confident the time needed to complete the training module is between (_____ , _____) minutes on average.

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

PRACTICE

A beverage company claims that on average, customers consume 500 mL of their new energy drink in a single sitting. To test this claim, a market researcher collects a random sample of 8 customers. Assume the sample is normal & make a 90% conf. int. for the true mean volume. Does the data support the company's claim?

Volume of Energy Drink Consumed (mL)							
480	495	470	490	505	475	460	495

HOW TO: Make a C.I. for Mean on TI-84

1) If given data, enter in **L1**

(STAT, **1:EDIT…**)

2) STAT > **TESTS**

3) If σ Known: **7: ZInterval**

If σ Unknown: **8: TInterval**

4)

If given data:	If given $\bar{x}, n, \ldots$:
DATA	**STATS**
σ: *(If Known)*	**σ:** *(If Known)* *or* **Sx:** *(If Unknown)*
List: L1	$\bar{x}$**:**
Freq: 1	**n:**
C-Level:	**C-Level:**

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

Topic Resource: Critical t-Values

Conf. Level	80%	90%	95%	98%	99%	99.9%
↓ df | $\alpha/2$ →	0.1	0.05	0.025	0.01	0.005	0.0005
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.924
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
...	...	...	...	...	...	...
40	1.303	1.684	2.021	2.423	2.704	3.551
50	1.299	1.676	2.009	2.403	2.678	3.496
60	1.296	1.671	2.000	2.390	2.660	3.460
70	1.294	1.667	1.994	2.381	2.648	3.435
80	1.292	1.664	1.990	2.374	2.639	3.416
90	1.291	1.662	1.987	2.368	2.632	3.402
100	1.290	1.660	1.984	2.364	2.626	3.390
...	...	...	...	...	...	...
∞	1.282	1.645	1.960	2.326	2.576	3.291