TOPIC: CHI-SQUARE GOODNESS OF FIT TEST

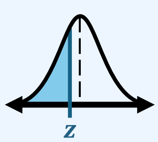
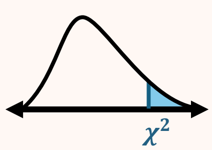
Goodness of Fit Test

- Use a **Goodness of Fit test** to determine if ____________ frequencies match "claimed" frequencies in a distribution.
  - Instead of testing just ***one*** parameter, you'll test the frequencies for ____________ categories.

EXAMPLE

You roll a 6-sided die 60 times and list the observed frequencies in the table. Determine if this die is fair by testing the goodness of fit of the die rolls with a uniform distribution. Use $\alpha = 0.05$.

Roll	1	2	3	4	5	6
Frequency	13	12	1	11	14	9

	Recall: Hypoth. Tests	New: Goodness of Fit Tests
1) Hypothesis	$H_0: \mu = \#$ $H_a: \mu$ </>/$\neq$ #	H_0: observed frequencies **match** claimed distribution H_a: observed frequencies **DO NOT match** claimed distribution, i.e. at least _________ of the probabilities is different from claimed.
2) Test Stat	$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$	*If claimed prob's same:* $\chi^2 = \sum \dfrac{(O-E)^2}{E}$ $E = \dfrac{n}{k}$ O = observed freq. E = expected freq. n = total sample size k = # of categories $n =$ ___ $k =$ ___ $E =$ ___ $\chi^2 = \dfrac{(\ \ -\ \ )^2}{\ } + \dfrac{(\ \ -10)^2}{10} + \dfrac{(\ \ -10)^2}{10} + \dfrac{(\ \ -10)^2}{10} + \dfrac{(\ \ -10)^2}{10} + \dfrac{(\ \ -10)^2}{10}$ $= \dfrac{\ }{10} + \dfrac{\ }{10} + \dfrac{\ }{10} + \dfrac{\ }{10} + \dfrac{\ }{10} + \dfrac{\ }{10}$
3) P-value		$df = k - 1$ $df =$ __  P-value = Area "beyond" χ^2 P-value = ______
4) Conclusion	Because P-value…	Because P-value **[< | >]** α, we **[REJECT | FAIL TO REJECT]** H_0. There is **[ENOUGH | NOT ENOUGH]** evidence to conclude that the observed frequencies do not match the claimed distribution, therefore it **[IS | IS NOT]** a good fit and the die _________ fair.
Criteria	Random samples? ☐ X is normal ***OR*** $n > 30$? ☐	Random Samples? ☐ Observed freq. for each category? ☐ $E \geq 5$ for each category? ☐

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PRACTICE

A gym owner wants to know if the gym has similar numbers of members across different age groups. The table shows the distribution of ages for members from a random survey. Write the null & alt. hypotheses to test the claim that the gym has equal numbers of members across all groups.

Age Group	18 – 25	26 – 35	36 – 45	46 – 55	56+
# of Members	54	46	53	49	48

H_0:

H_a:

Find the χ^2 statistic to test the claim that the gym has equal numbers of members of all age ranges.

$n =$ ___ $k =$ ___

$E =$ ___ $\chi^2 =$

Recall
$E = \frac{n}{k}$

(Expected Freq. – Uniform)

Recall
$\chi^2 = \sum \frac{(O-E)^2}{E}$

Using $\chi^2 = 0.92$ & $\alpha = 0.05$, test the claim that the gym has equal numbers of members of all age ranges.

$df =$ __

Recall
$df = k - 1$

P-value = _______

Because P-value **[< | >]** α, we **[REJECT | FAIL TO REJECT]** H_0.

There is **[ENOUGH | NOT ENOUGH]** evidence to conclude that the # of members is significantly different for at least one of the age groups at this gym.

So the claimed dist. **[IS | IS NOT]** a good fit.

Does this data set fit the criteria for a G.O.F. test?

Random Samples? ☐

Observed freq. for each category? ☐

$E \geq 5$ for each category? ☐

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EXAMPLE

A company runs a customer satisfaction survey where customers rate their experience. The manager claims that responses will NOT be the same across all 5 rating categories. A random sample of 100 survey responses is collected with the following observed frequencies. Using $\alpha = 0.05$, test the manager's claim.

Survey Response	Very Poor	Poor	Neutral	Good	Very Good
Frequency	13	14	26	29	18

Recall

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

$$df = k - 1$$

Random Samples? ☐

Observed freq. for each category? ☐

$E \geq 5$ for each category? ☐

H_0:

H_a:

$n =$ ___ $k =$ ___ $E =$ ___

Recall

$$E = \frac{n}{k}$$

(Expected Freq. – Uniform)

Because P-value **[< | >]** α, we **[REJECT | FAIL TO REJECT]** H_0.

There is **[ENOUGH | NOT ENOUGH]** evidence of H_a.

So the claimed dist. **[IS | IS NOT]** a good fit.

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Goodness of Fit Test: Unequal Probabilities

◆ If claimed probabilities AREN'T equal, find expected freq's using *given probabilities* instead of # of categories.

EXAMPLE

According to Benford's Law, the 1st digits of large numbers in real-world data sets follow the probability distribution in the table. Also listed in the table are the 1st-digit frequencies of the populations of 100 random cities across Europe. Find the test statistic that would be used to test if these digits follow Benford's Law.

1st Digit	1	2	3	4	5	6	7	8	9
Frequency (O)	18	14	11	13	12	9	5	8	10
Benford's Law Prob's (p)	.301	.176	.125	.097	.079	.067	.058	.051	.046
Exp. Freq. (E)									
$\frac{(O-E)^2}{E}$									

New

$E = np$ $n =$ ____

Recall

$\chi^2 = \sum \frac{(O-E)^2}{E}$

$\chi^2 =$ _____

PRACTICE

A marketing associate for a supermarket chain wants to determine how many of each snack type to cook. According to previous market research, customers' preferences tend to follow the distribution in the table. If approximately 200 snack items are purchased in a day, what is the expected frequency of each snack type?

1st Digit	Chips	Cookies	Crackers	Nuts	Granola Bars
Preferences from Research	36%	21%	12%	8%	23%
Expected Sales					

Recall

$E = \frac{n}{k}$ *(claimed prob's SAME)*

$E = np$ *(claimed prob's DIFF)*

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EXAMPLE

A regional airline company classifies its customers into three types based on ticket purchase: Business (50%), Leisure (35%), & Last-minute (15%). The marketing department wants to verify whether this distribution still holds after changes to the airline's pricing model. A random sample of 200 recent ticket purchases is analyzed with the observed counts listed in the table. Use a goodness of fit test and $\alpha = 0.10$ to determine if the distribution fits.

Customer	Business	Leisure	Last-Min.
Obs. Freq.	98	71	31

$n =$____

$df =$____

Recall

$E = \frac{n}{k}$ *(claimed prob's SAME)*

$E = np$ *(claimed prob's DIFF)*

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$df = k - 1$$

(G.O.F. Test)

Random Samples? ☐

Observed freq. for each category? ☐

$E \geq 5$ for each category? ☐

H_0:

H_a:

Because P-value **[< | >]** α, we **[REJECT | FAIL TO REJECT]** H_0.

There is **[ENOUGH | NOT ENOUGH]** evidence of H_a.

So the claimed dist. **[IS | IS NOT]** a good fit.