Constructing Confidence Intervals for Proportions

lacktriangle To make a conf. int. for p, use point estimator \hat{p} & margin of error:

New
$$E = z_{\alpha/2} \cdot \sqrt{}$$

► Recall: The sampling distribution of $\hat{p} = \frac{x}{n}$ is approximately normal when $np \ge 5$ and $nq \ge 5$.

EXAMPLE

From a survey of 200 people, 90 preferred computers from Brand A over Brand B. Construct a 90% confidence interval for the true (population) proportion of people who prefer computers from Brand A.

We are ____ % confident that the true proportion of people who prefer Brand A computers is between _____ & ____

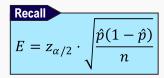
HOW TO: Make a Confidence Interval for p

- 1) Verify # of successes ≥ 5
 - **AND** # of failures ≥ 5
- **2)** Find critical value: $z_{\alpha/2}$
- 3) If not given: $\hat{p} = \frac{x}{n}$
- 4) Margin of Error: $E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
- 5) Find upper & lower bounds

$$(\hat{p} - E, \hat{p} + E)$$

PRACTICE

A factory manager wants to estimate the proportion of defective items produced. In a batch of 20 items, the factory has produced 6 with defects. Find the margin of error for a 98% confidence interval for the true proportion of defective items.



PRACTICE

Over the first 20 days of the semester, one student is late to class on 6 days. Construct a 98% confidence interval for the true proportion of time this student is late.

HOW TO: Make a Confidence Interval for p

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- **4)** Margin of Error: $E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
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$$(\hat{p} - E, \hat{p} + E)$$

EXAMPLE

Apple conducted a survey to see what percent of people preferred the iPhone over all other types of smart phones. They found that 62% of the 100 sampled individuals preferred the iPhone. Construct (A) a 99% confidence interval &(B) a 92% confidence interval for the proportion of individuals who prefer the iPhone. (C) Why are these two confidence intervals different, even though they are based on the same sample information?.

HOW TO: Make a Confidence Interval for p

1) Verify # of successes ≥ 5

AND # of failures ≥ 5

2) Find critical value: $z_{\alpha/2}$

3) If not given: $\hat{p} = \frac{x}{n}$

4) Margin of Error: $E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

5) Find upper & lower bounds

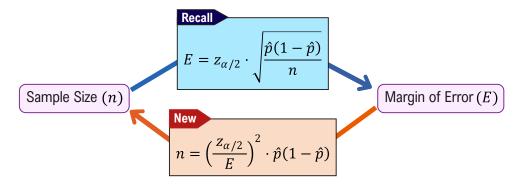
$$(\hat{p} - E, \hat{p} + E)$$

EXAMPLE

For a fair pair of dice, the probability of getting snake eyes (two ones) is 1/36. You have a pair of dice that could be weighted to 1's more often. You roll the dice 50 times and get snake eyes on 11 of the rolls. (\mathbf{A}) Create a 99% confidence interval for the true proportion of snake eye rolls. (\mathbf{B}) Is 1/36 in the interval? (\mathbf{C}) Are the dice weighted?

Finding the Minimum Sample Size Needed for a Confidence Interval

◆ When you're not given the sample size, *n*, you'll need to determine it based on the given margin of error.



- ►We can *rearrange* the margin of error eqn. for *n*, plug in values, & round UP to next _____ number.
- If you're not given \hat{p} , use $\hat{p} = \underline{\hspace{1cm}}$.

EXAMPLE

You're running for student body president. You want to estimate the proportion of people who would vote for you with 90% confidence. You also want the margin of error to be .02 or smaller. How many students are needed in your sample?

PRACTICE

A previous study found that your school consists of 60% White/Caucasian students. You want the 98% confidence interval for the proportion of White/Caucasian students to be no more than .05 away from the true proportion. How many students must you include in a sample to create this confidence interval?

PRACTICE

A labor economist wants to estimate, with 90% confidence, the proportion of remote workers in the workforce. The economist wants the estimate to be accurate within 4% of the true population proportion. What is the minimum sample size needed for this estimate?

Recall
$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 \cdot \hat{p}(1-\hat{p})$$

PRACTICE

Your company has asked you to estimate the proportion of people who prefer the color red over other primary colors for manufacturing purposes. If they want the estimate to be within .01 of the true proportion with 95% confidence, how many people should you survey?

PRACTICE

You want to make a 97% confidence interval for the population proportion of people between 20-30 years old who have gotten a speeding ticket in the past 2 years. A prior study found that 26% of people between 20-30 years old have received a speeding ticket in the last year. If you want your estimate to be accurate within 3% of the true population proportion, what is the minimum sample size needed?

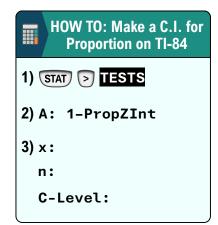
C.I. Using TI-84 - Proportion

◆ To make a C.I. for the population proportion using a calculator, use the **1-PropZInterval** function.

EXAMPLE

A bakery is researching flavor preferences & takes a random sample of 50 customers. 32 said chocolate chip is their favorite. Make a 90% conf. int. for the proportion of customers who prefer chocolate chip cookies.

We are _____ % confident that the proportion of customers preferring chocolate chip cookies is between (_____ , ____).



PRACTICE

Make a confidence interval for p given the following values.

(A)
$$x = 314, n = 500, C = 99\%$$

$$\hat{p} = 0.15, n = 75, C = 90\%$$



PRACTICE

An economist is evaluating how frequently the U.S. inflation rate exceeds the Federal Reserve's long-term target of 2% per yr $\approx 0.17\%$ per month. The economist finds that in 34 of the 48 sampled months, the monthly inflation rate did exceed 0.17%.

(A) Make a 95% confidence interval for the true proportion of months in which the inflation rate exceeds the target.

We are _____ % confident the inflation rate exceeds the target in between (_____ , ____) of months.

(*B*) Under stable conditions the inflation rate should not exceed the target more than 20% of the time. Can the economist conclude that inflation has exceeded the target more than 20%?

