TOPIC: DIFFERENTIALS

Finding Differentials

◆ For a function $y = f(x)$, where $f'(x) = \frac{dy}{dx}$, dy and dx are called **differentials**.

EXAMPLE Given the function $f(x) = x^3 + x$, find dy when $x = 2$ and $dx = 0.1$.

New **Differentials**

$\frac{dy}{dx} = f'(x) =$ _____

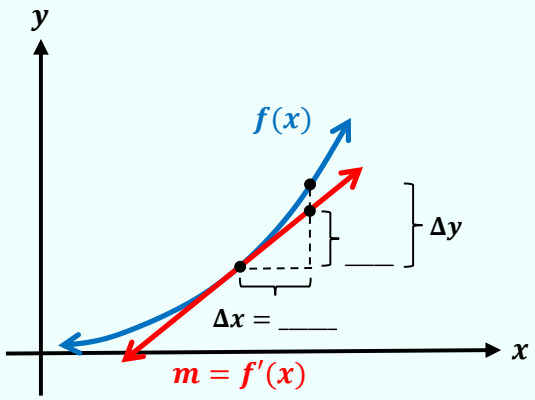
$dy =$ _____

$dy =$ _____

$dy =$ _____

$dy =$ _____

EXAMPLE Given the function $f(x) = x^3 + x$, use differentials, with $x = 2$ and $dx = 0.1$, to estimate $f(2.1)$.



$f(x + \_\_\_\_\_) \approx f(x) + \_\_\_\_\_$

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PRACTICE If $f(x) = x^3 - 8x + 6$, find the differential dy when $x = 2$ and $dx = 0.2$.

PRACTICE If $f(x) = \sqrt{x + 2}$, find the differential dy when $x = 2$ and $dx = 0.01$.

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EXAMPLE Given the equation $f(x) = x^3 - 2x$, ($\boldsymbol{A}$) find dy when $x = 2$ and $dx = 0.2$, and ($\boldsymbol{B}$) estimate $f(2.2)$.

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Error in Differential Approximation

◆ Recall: We can use differentials to approximate the value of the function. $\boxed{f_{approx} = f(x + dx) \approx f(x) + dy.}$

► **Absolute error** is the **[DIFFERENCE | RATIO]** between the _____ value and the _____ value.

► **Relative error** is the **[DIFFERENCE | RATIO]** between the **absolute error** and the _____ value.

EXAMPLE Given $f(x) = x^3 + x$, use differentials to estimate $f(2.1)$, then calculate the absolute and relative error in your approximation.

Absolute Error	Relative Error
$\boxed{abs\ error = \lvert f_{exact} - f_{approx} \rvert}$	$\boxed{rel\ error = \dfrac{abs\ error}{f_{exact}}}$
$f_{approx} = 11.3$	
$f_{exact} = f(2.1) =$ _____	$f_{exact} = f(2.1) = 11.361$
$abs\ error =$ _____	$rel\ error =$ _____
$abs\ error =$ _____	$rel\ error \approx$ _____

◆ The **percentage error** is just the relative error multiplied by 100. $\boxed{\%\ error = rel\ error \cdot 100\%}$

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EXAMPLE

Given $f(x) = 8x - x^2$, if $x = 3$ and $dx = 0.1$, use differentials to estimate $f(3.1)$, then calculate the absolute and relative error in your approximation.