TOPIC: MOTION ANALYSIS

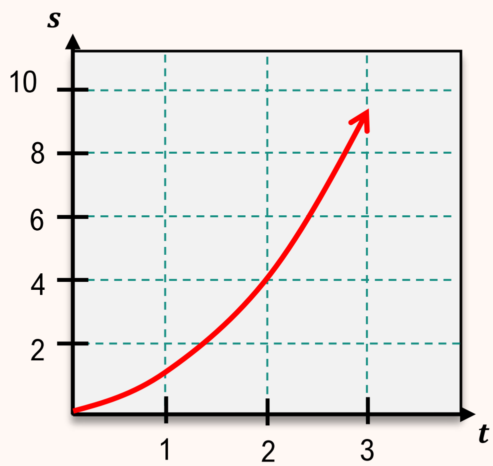
Derivatives Applied to Velocity

◆ The motion of an object over time is often described using its position function ______ & velocity function ______.

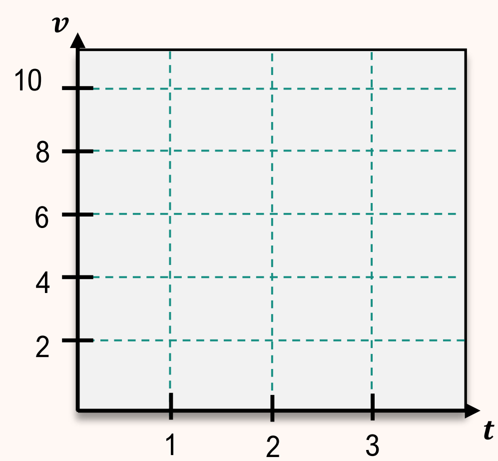
► Velocity = change in position (displacement) over time: $v(t) = \frac{\quad}{\quad}$

EXAMPLE Given an object's position $s(t)$ in meters, find the missing values based on the time interval $0\text{s} \leq t \leq 2\text{s}$.

New **Using Derivatives to Solve Motion Problems**



$s(t) = t^2$



$v(t) = \_\_\_\_$

$v(2) = \_\_\_\_$

(Instantaneous Velocity)

$\Delta s = s(t_{final}) - s(t_{initial})$

(Displacement)

$\Delta s = \_\_\_\_ - \_\_\_\_$

$= \_\_\_\_ - \_\_\_\_$

$= \_\_\_\_ - \_\_\_\_$

$v_{avg} = \frac{\quad}{\quad}$

(Average Velocity)

$v_{avg} = \frac{\quad}{\quad}$

$= \_\_\_$

$Speed = \quad v(t)$

(Speed)

$speed = |\_\_\_\_|$

$= \_\_\_$

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PRACTICE Given the position equation $s(t)$, calculate the *average* velocity (in meters per second) based on the given time interval, and the *instantaneous* velocity (in meters per second) at the end of the time interval.

$$s(t) = 9t - t^2, \quad 0 \leq t \leq 3$$

PRACTICE Given the position equation $s(t)$, calculate the *average* velocity (in meters per second) based on the given time interval, and the *instantaneous* velocity (in meters per second) at the end of the time interval.

$$s(t) = \frac{30}{t+5}, \qquad -4 \leq t \leq 0$$

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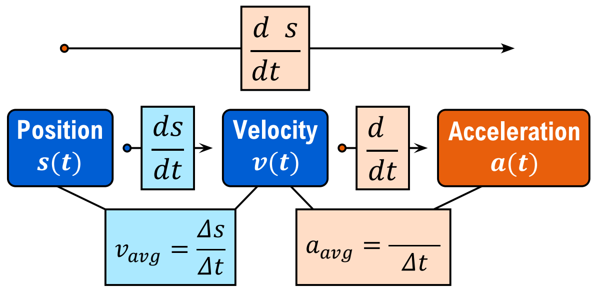
EXAMPLE

The height (in meters) of a projectile shot vertically upward from a point $5\ m$ above ground level with an initial velocity of $20\frac{m}{s}$ is $h(t) = 5 + 20t - 4.9t^2$. $(\boldsymbol{A})$ What is the vertical velocity at $t = 3$ seconds? $(\boldsymbol{B})$ When does the projectile reach its maximum height? $(\boldsymbol{C})$ What is the maximum height of the projectile?

TOPIC: MOTION ANALYSIS

Derivatives Applied to Acceleration

◆ Acceleration (a) is the *change* in velocity over time, i.e. the _________ deriv. of $v(t)$ and _________ deriv. of $s(t)$.



EXAMPLE Given the velocity function $v(t) = t^3 - 3t^2 + 2t$ in meters per second, answer the following based on the time interval $0\text{s} \leq t \leq 3\text{s}$.

$(\boldsymbol{A})$ What is the change in velocity?

$\Delta v =$ _____

$(\boldsymbol{B})$ What is the average acceleration?

$a_{avg} =$ _____

$(\boldsymbol{C})$ What is the instantaneous acceleration at the end of the time interval?

$a(\_\_) =$ _____

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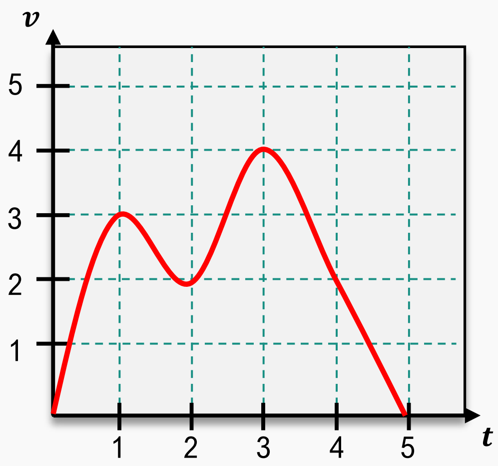
PRACTICE Given the position of an object $s(t) = 12t - t^2$ (in meters) find the acceleration of the object at $t = 5$ seconds.

PRACTICE Given below is the graph of *velocity* with respect to time. At which time(s) would *acceleration* be 0?



TOPIC: MOTION ANALYSIS

EXAMPLE

The height (in meters) of a projectile shot vertically upward from ground level with an initial velocity of $15\,\frac{m}{s}$ is $h(t) = 15t - 4.9t^2$. $(\boldsymbol{A})$ What is the vertical velocity at $t = 3$ seconds? $(\boldsymbol{B})$ What is the vertical acceleration at $t = 3$ seconds? $(\boldsymbol{C})$ If the maximum height of the projectile is $11.5\ m$, when will the projectile initially reach half of its maximum height?