TOPIC: CURVE SKETCHING

Summary of Curve Sketching

◆ We can use f' & f'' along with basic graphing knowledge to sketch the graph of f.

EXAMPLE Sketch the graph of $f(x) = x^3 - 3x^2 + 4$.

HOW TO: Sketch Graph of f Using f' & f''

1) Determine **domain** of f: _____

2) Find x & y **intercepts**:

$f(x) = 0$ $\qquad$ $f(0) = 0^3 - 3(0)^2 + 4$

$(x-2)^2(x+1) = 0$

3) If needed, find **asymptotes**

4) Symmetry → $f(-x) =$ $\begin{bmatrix} f(x) & | & -f(x) \\ y\text{-axis} & | & \text{origin} \end{bmatrix}$

$(-x)^3 - 3(-x)^2 + 4 =$

5) a. Inc/Dec → Find crit. pts: $f'(x) = 0$ / DNE

$f'(x) = 3x^2 - 6x = 3x(x-2)$ →

f' $\quad$ + $\quad$ − $\quad$ +

$\longleftrightarrow$ x

f''

b. Concave up/down → Inf. pts: $f''(x) = 0$ / DNE

$f''(x) = 6x - 6 = 6(x-1)$ →

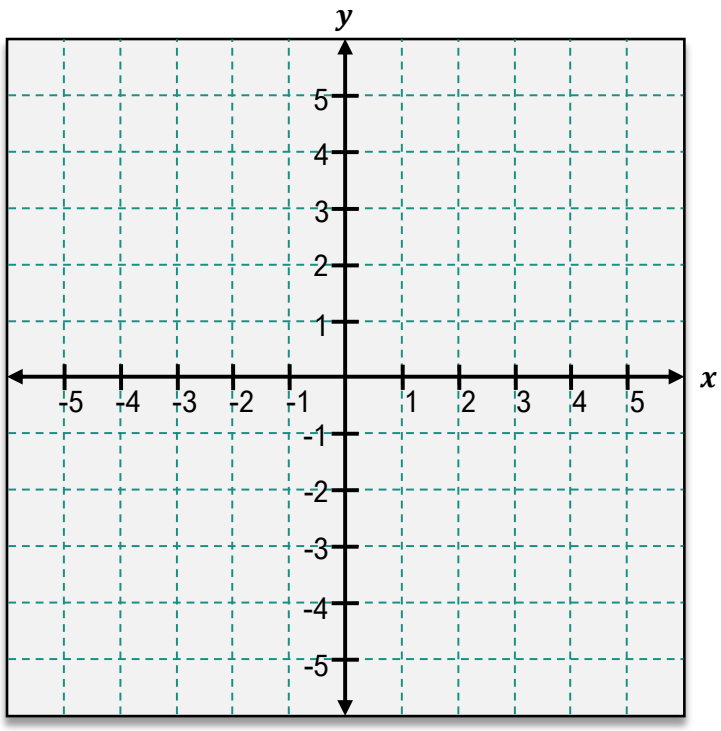
f'' $\quad$ − $\quad$ +

6) Local Extrema → Use **(5a)**: + to −, local **MAX**

$f(\_\_\_) = \_\_\_\_$ $\qquad$ − to +, local **MIN**

$f(\_\_\_) = \_\_\_\_$

7) Connect points with smooth curve based on 4-6



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EXAMPLE Sketch the graph of $f(x) = \frac{x-1}{x+2}$.

HOW TO: Sketch Graph of f Using f' & f''

1) Determine **domain** of f: _____

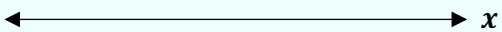
2) Find x & y **intercepts**: $f(x) = 0$ & $f(0)$

3) If needed, find **asymptotes**

4) Symmetry → $f(-x) =$ $\begin{bmatrix} f(x) \\ y\text{-axis} \end{bmatrix} \begin{vmatrix} -f(x) \\ \text{origin} \end{vmatrix}$ sym. about

5) a. Inc/Dec → Find crit. pts: $f'(x) = 0$ / DNE

f' (+ / −)



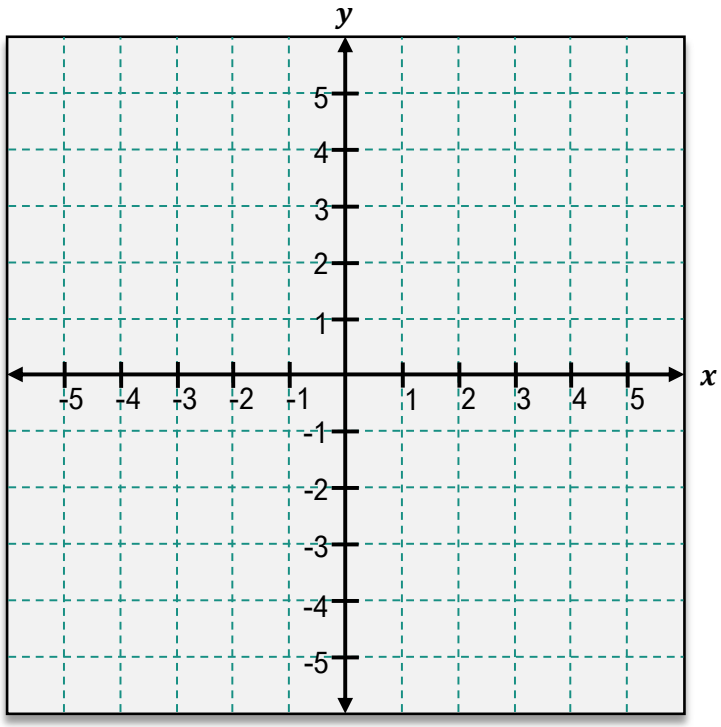
f''

b. Concave up/down → Inf. pts: $f''(x) = 0$ / DNE

6) Local Extrema → Use **(5a)**: + to −, local **MAX**

− to +, local **MIN**

7) Connect points with smooth curve based on 4-6



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EXAMPLE Sketch the graph of $g(x) = \sqrt{x^3 + 8}$.

HOW TO: Sketch Graph of f Using f' & f''

1) Determine **domain** of f: _____

2) Find x & y **intercepts**: $f(x) = 0$ & $f(0)$

3) If needed, find **asymptotes**

4) Symmetry → $f(-x) =$ $\begin{bmatrix} f(x) & -f(x) \\ y\text{-axis} & \text{origin} \end{bmatrix}$ sym. about

5) a. Inc/Dec → Find crit. pts: $f'(x) = 0$ / DNE

f' (+ / −)

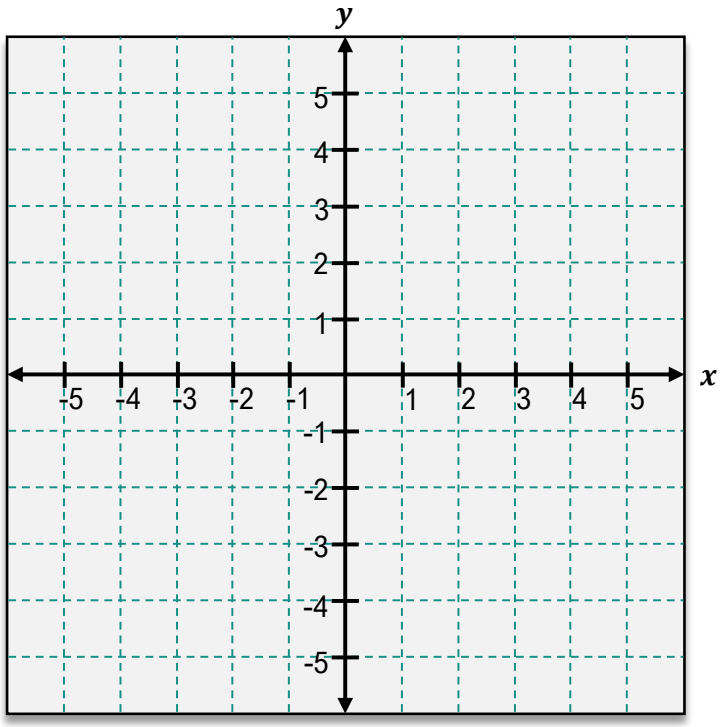
f''

b. Concave up/down → Inf. pts: $f''(x) = 0$ / DNE

6) Local Extrema → Use **(5a)**: + to −, local **MAX**

− to +, local **MIN**

7) Connect points with smooth curve based on 4-6



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EXAMPLE Sketch a graph of the function f with the following properties.

- ☐ Continuous & differentiable everywhere
- ☐ x-intercepts at $x = 0$ and $x = 2$
- ☐ $f'(x) > 0$ on $(-\infty, 1)$ and $(2, \infty)$
- ☐ $f'(x) < 0$ on $(1, 2)$
- ☐ $f''(x) > 0$ for $x < 0.5$ & $x > 1.5$
- ☐ $f''(x) < 0$ for $0.5 < x < 1.5$