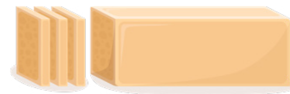
TOPIC: INTRODUCTION TO VOLUME & THE DISK METHOD

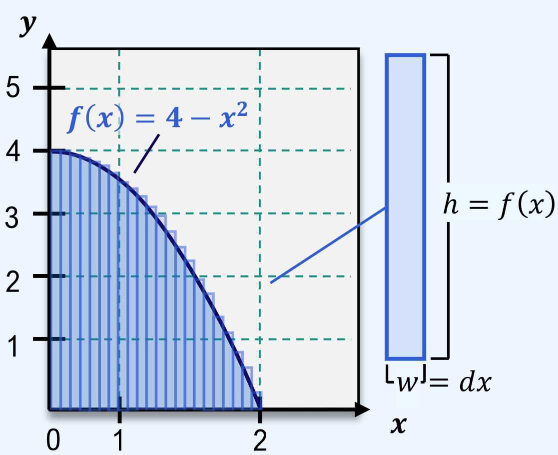
Introduction to Cross Sections



- Recall: To find area, add the area of many small rectangles. To find volume, add the volume of many thin slices.
  - ► Volume of slice = Area of Cross Section • width. A **cross section** is a 2D shape we get from cutting a 3D solid.

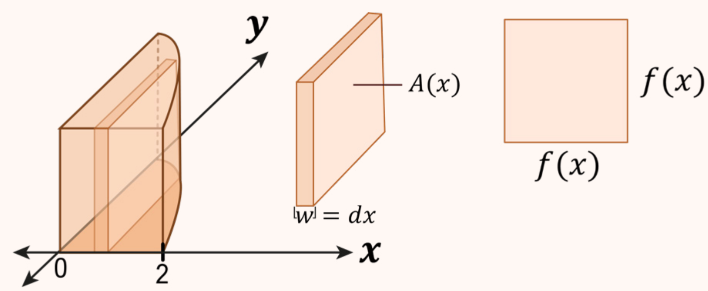
EXAMPLE Set up an int. for the volume of a solid with base $f(x) = 4 - x^2$ on $[0, 2]$ with square cross sections.

Recall — **Integrals for Area**



$$Area = \int_a^b f(x)dx$$

New — **Integrals for Volume**

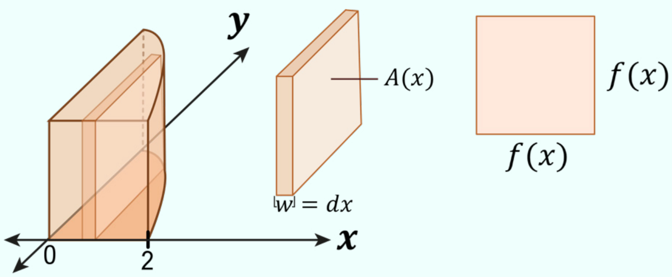


$$Volume = \int_a^b A(x)dx$$

- *ALWAYS* sketch the solid and a front facing cross section.

TOPIC: INTRODUCTION TO VOLUME & THE DISK METHOD

EXAMPLE Find the volume of a solid with base $f(x) = 4 - x^2$ on $[0, 2]$, with square cross sections.



Recall

$$Volume = \int_a^b A(x)dx$$

$$Volume = \int_0^2 (4 - x^2)^2 \, dx$$

TOPIC: INTRODUCTION TO VOLUME & THE DISK METHOD

PRACTICE

Find the volume of the solid whose base is the region bounded by the function $f(x) = \sqrt{9 - x^2}$ and the x-axis with square cross sections perpendicular to the x-axis.

Recall

$$Volume = \int_a^b A(x)dx$$

TOPIC: INTRODUCTION TO VOLUME & THE DISK METHOD

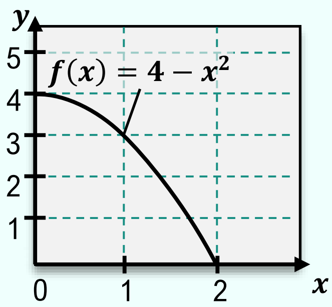
EXAMPLE

Set up an integral for the volume of a solid with base: $y = 4 - x^2$ on $[0,2]$ & cross sections in the following shapes, parallel to the y-axis.

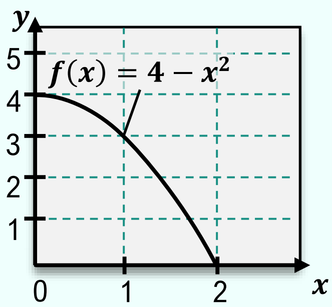
(A) CS = **Isosceles Right Triangles**

Recall

$$Volume = \int_a^b A(x)dx$$



(B) CS = **Circles**



TOPIC: INTRODUCTION TO VOLUME & THE DISK METHOD

PRACTICE

Find the volume for a solid whose base is the region between the curve $y = \sqrt{\sin x}$ and the x-axis on the interval from $[0, \pi]$ and whose cross sections are equilateral triangles with bases parallel to the y-axis.

Recall

$$Volume = \int_a^b A(x)dx$$

TOPIC: INTRODUCTION TO VOLUME & THE DISK METHOD

Finding Volume Using Disks

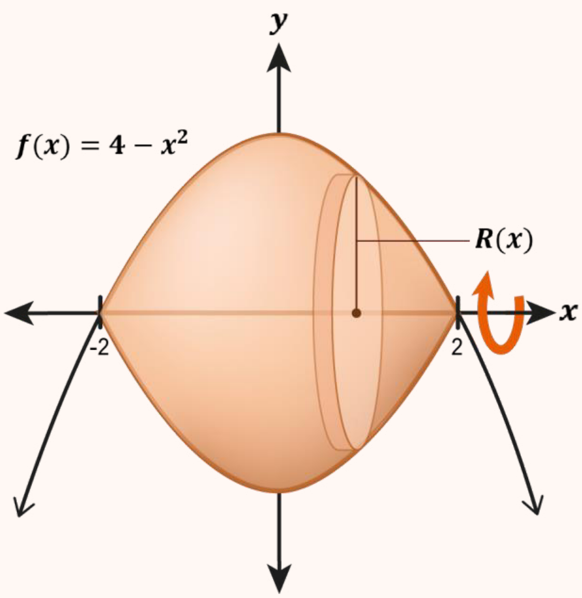
Recall: $Volume = \int_a^b A(x)dx$

- Recall: To find volume of a 3D solid, integrate the area function of a cross section.
  - A **Solid of Revolution** is a solid formed by revolving a curve around an axis (x-axis, y-axis, $y = 2$, etc.).
  - The cross sections of a solid of rev. are _________ (*disks*) whose radius = the dist. from the _______ to the _____.

EXAMPLE Set up an int. for the volume of a solid formed by rotating $f(x) = 4 - x^2$ on $[-2,2]$ about the x-axis.

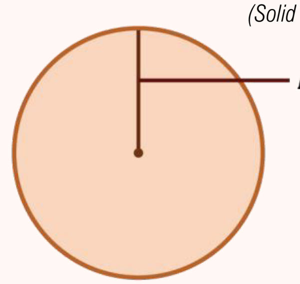
New **Solids of Revolution: x — Axis**

Recall: $Area\ of\ circle = \pi r^2$



$Volume = \int_a^b$ _________ dx

(Solid of Revolution – Disk Method)



$R(x)$ = curve – axis

= __________

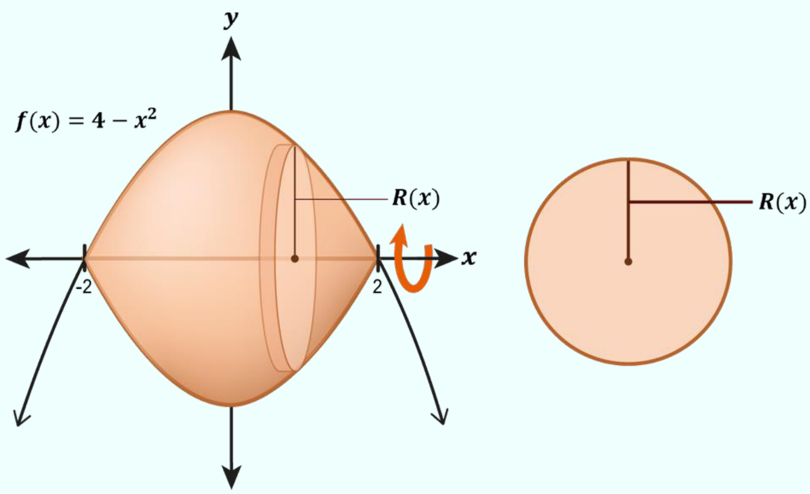
= __________

= __________

$Volume =$

TOPIC: INTRODUCTION TO VOLUME & THE DISK METHOD

EXAMPLE Find the volume of the solid formed by rotating $f(x) = 4 - x^2$ on $[-2,2]$ about the x-axis.



Recall

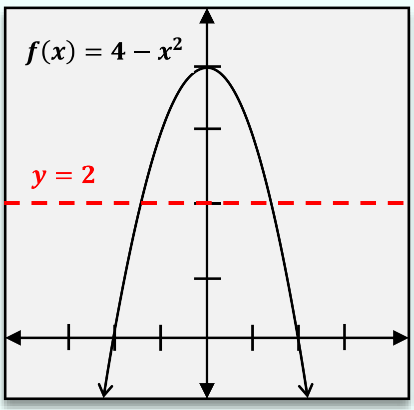
$$Volume = \int_a^b \pi[R(x)]^2 dx$$

$$Volume = \int_{-2}^{2} \pi(4 - x^2)^2 dx$$

TOPIC: INTRODUCTION TO VOLUME & THE DISK METHOD

EXAMPLE

Set up an integral that represents the volume of the solid of revolution formed by rotating the area between $f(x) = 4 - x^2$ & $y = 2$ around the line $y = 2$.

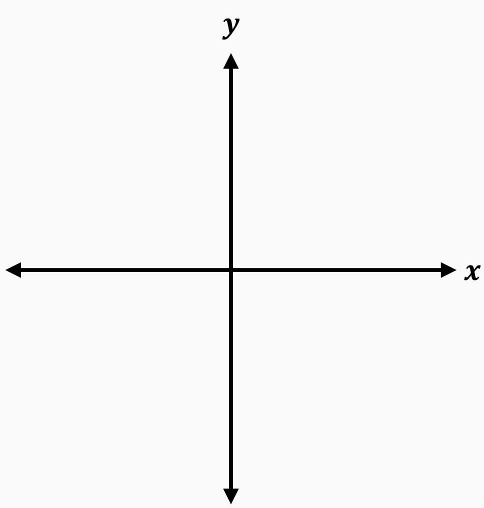


Recall

$$Volume = \int_a^b \pi[R(x)]^2 dx$$

PRACTICE

Find the volume of the solid obtained by rotating the region bounded by $y = x + 4$, $y = 0$, $x = 1$ & $x = 5$ about the x-axis.



TOPIC: INTRODUCTION TO VOLUME & THE DISK METHOD

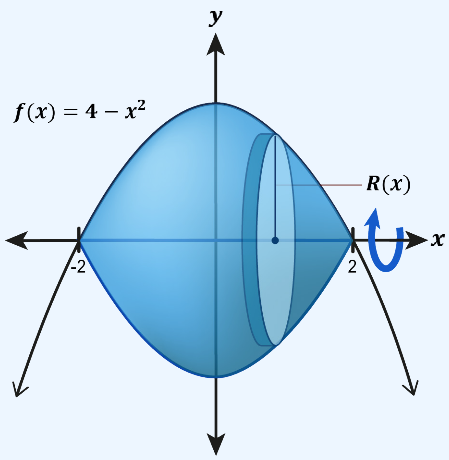
Disk Method Using y-Axis

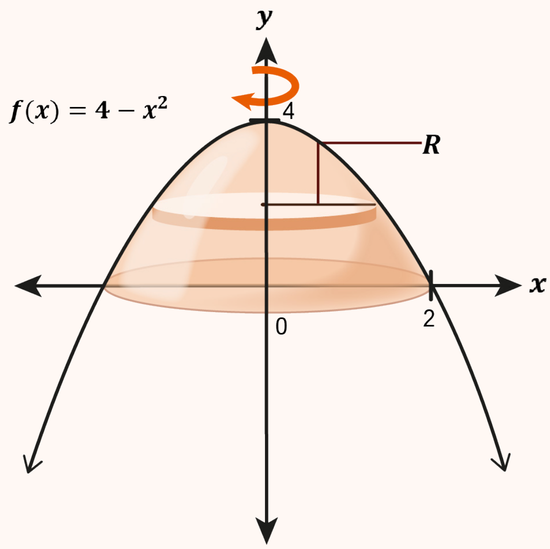
◆ Recall: To find volume of a solid of revolution, integrate the area function of circular cross section.

► When revolving around y-axis, **1)** write fcn in terms of ____, **2)** find ______ & __________, **3)** integrate w.r.t. ______.

EXAMPLE Set up an int. for the volume of a solid formed by rotating $f(x) = 4 - x^2$ on $[0,2]$ about the y-axis.

Recall Revolve around x – Axis	**New** Revolve around y – Axis
$Volume = \int_a^b \pi[R(x)]^2 dx$	$Volume = \int_c^d \pi[R(\_\_)]^2 d\_\_$





$R(\_\_)$ = curve – axis

= __________

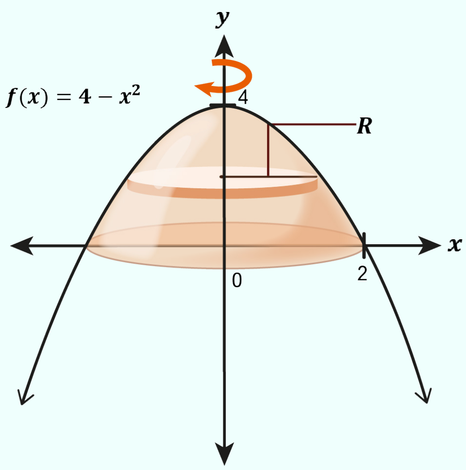
= __________

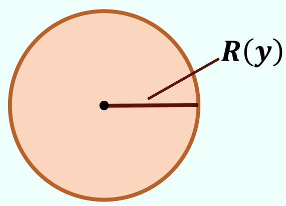
= __________

$Volume =$

TOPIC: INTRODUCTION TO VOLUME & THE DISK METHOD

EXAMPLE Find the volume of a solid formed by rotating $f(x) = 4 - x^2$ on $[0,2]$ about the y-axis.





Recall

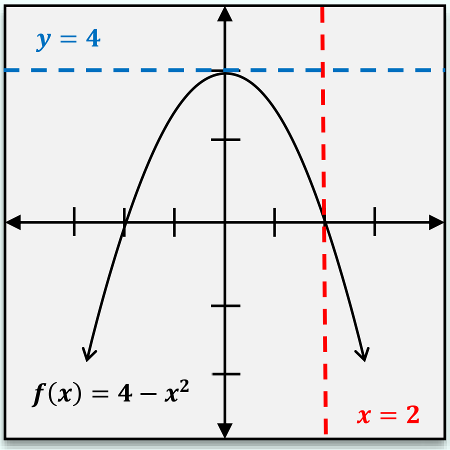
$$Volume = \int_c^d \pi[R(y)]^2 dy$$

$$Volume = \int_0^4 \pi\left(\sqrt{4-y}\right)^2 dy$$

TOPIC: INTRODUCTION TO VOLUME & THE DISK METHOD

EXAMPLE

Find the volume of the solid of revolution formed by rotating the area between $f(x) = 4 - x^2$, $y = 4$, & $x = 2$ in Q1 around the line $x = 2$.



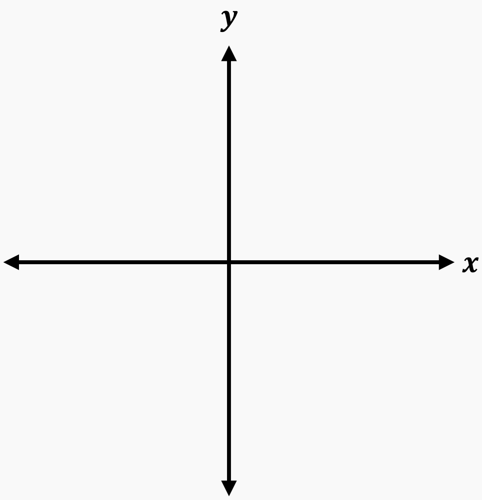
Recall

$$Volume = \int_c^d \pi[R(y)]^2 dy$$

TOPIC: INTRODUCTION TO VOLUME & THE DISK METHOD

PRACTICE

Find the volume of the solid formed by revolving the area bounded by $f(x) = x^2$ from $x = 0$ to $x = 3$ and the y-axis around the y-axis.



Recall

$$Volume = \int_c^d \pi[R(y)]^2 dy$$