TOPIC: THE FIRST DERIVATIVE TEST

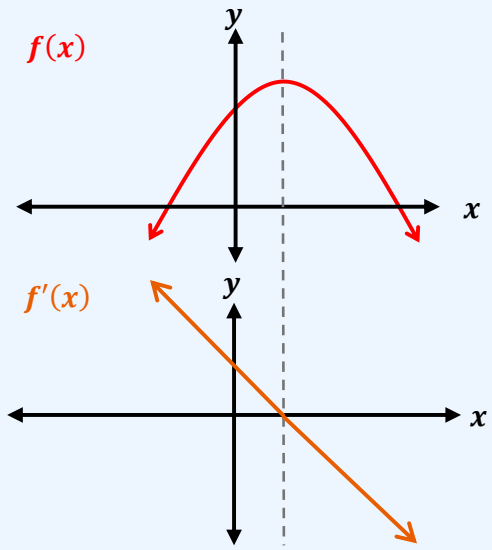
Determining Where a Function Is Increasing & Decreasing

◆ Recall: Looking at a graph from left to right, a function is increasing if it goes up & decreasing if it goes down.

► Given a function, find where it is **increasing** or **decreasing** based on the _____ of the _____.

EXAMPLE Using the derivative, determine if $f(x)$ is increasing or decreasing at $x = 0$ & $x = 5$.

Recall Increasing/Decreasing Graphically



New Increasing/Decreasing Using Derivative

$$f(x) = -x^2 + 4x + 5$$

$x = 0$:

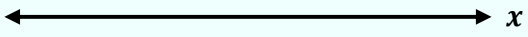
If f' is **[+ | −]**, f is **[INC | DEC]**

$x = 5$:

If f' is **[+ | −]**, f is **[INC | DEC]**

EXAMPLE Determine the intervals for which $f(x)$ is increasing or decreasing.

$$f(x) = -x^2 + 4x + 5$$



HOW TO: Determine Intervals of Increase & Decrease

1) Find critical points:
$f'(x) = 0$ or $f'(x)$ DNE

2) Make sign chart _____ based on critical points

3) Plug value from each int. into f':
If $+$, f _____ on interval
If $-$, f _____ on interval

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PRACTICE Identify the open intervals on which the function is increasing or decreasing.

(A)

$f(x) = 3x^4 + 8x^3 - 18x^2 + 7$

HOW TO: Determine Intervals of Increase & Decrease

1) Find critical points:
$f'(x) = 0$ or $f'(x)$ DNE

2) Make sign chart **intervals** based on critical points

3) Plug value from each int. into f':
If $+$, f **INC** on interval
If $-$, f **DEC** on interval

(B)

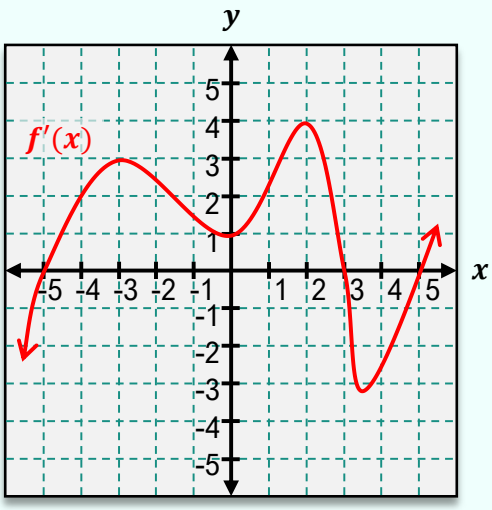
$f(x) = x^{2/3}\,(4 - x)$

(C)

$f(x) = \sin^2 x$ on $[0, \pi]$

TOPIC: THE FIRST DERIVATIVE TEST

EXAMPLE Identify the open intervals where $f(x)$ is increasing or decreasing based on the graph of $f'(x)$.



TOPIC: THE FIRST DERIVATIVE TEST

The First Derivative Test: Finding Local Extrema

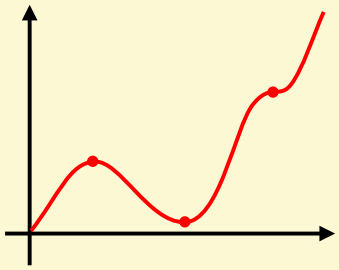
◆ Recall: The sign of the derivative tells us whether a function is increasing (f' is $+$) or decreasing (f' is $-$).

► Local extrema occur where the sign of the derivative f' _____.

First Derivative Test

Suppose c is a critical point of a continuous function f. If at c,

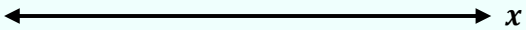
- f' changes from ___→___, f has a local [**MAX** | **MIN**] at c.
- f' changes from ___→___, f has a local [**MAX** | **MIN**] at c.
- f' does _____ change sign, f has ____ local extrema at c.



EXAMPLE Locate the local extrema of the function $f(x) = x^3 - 3x^2 + 4$.

$$f'(x) = 3x^2 - 6x$$

$$= 3x(x - 2)$$



HOW TO: Find Local Extrema Using First Derivative Test

1) Find critical points:
$f'(x) = 0$ or $f'(x)$ DNE

2) Make sign chart intervals based on critical points

3) Plug value from each int. into f'
If f' changes from:
$+ \to -$, crit. pt. is local **MAX**
$- \to +$, crit. pt. is local **MIN**

4) If asked: Find **value** of max/min by plugging crit. pt. into _____

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EXAMPLE Identify the local minimum and maximum values of the given function, if any.

(A)

$$f(x) = (x+7)^3$$

HOW TO: Find Local Extrema Using First Derivative Test

1) Find critical points:

$f'(x) = 0$ or $f'(x)$ DNE

2) Make sign chart intervals based on critical points

3) Plug value from each int. into f'

If f' changes from:

$+ \to -$, crit. pt. is local **MAX**

$- \to +$, crit. pt. is local **MIN**

4) If asked: Find **value** of max/min by plugging crit. pt. into $f(x)$

(B)

$$h(t) = \frac{t^3}{2t+1}$$

TOPIC: THE FIRST DERIVATIVE TEST

PRACTICE Identify the local minimum and maximum values of the given function, if any.

$f(\theta) = \sin\theta + \cos^2\theta$ on $[0, \pi]$

HOW TO: Find Local Extrema Using First Derivative Test

1) Find critical points:

$f'(x) = 0$ or $f'(x)$ DNE

2) Make sign chart intervals based on critical points

3) Plug value from each int. into f'

If f' changes from:

$+ \to -$, crit. pt. is local **MAX**

$- \to +$, crit. pt. is local **MIN**

4) If asked: Find **value** of max/min by plugging crit. pt. into $f(x)$

EXAMPLE Identify the global and local minimum and maximum values for the given function, if any.

$f(x) = x^2 + 4x + 17$