MASTER TABLE: TRIG IDENTITIES

◆ **NOTE**: This table spans multiple videos.

TRIG IDENTITIES			
Name	**Identity**	**Example**	**Use when…**
Reciprocal	$\csc\theta = \frac{1}{\sin\theta}$	$\sec\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\left(\frac{1}{2}\right)} = 2$	You need to rewrite an expression in terms of $\sin$ & $\cos$
	$\sec\theta = \frac{1}{\cos\theta}$		
	$\cot\theta = \frac{1}{\tan\theta}$		
Quotient	$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\tan\frac{\pi}{4} = \frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)}$	
	$\cot\theta = \frac{\cos\theta}{\sin\theta}$		
Even – Odd	$\cos(-\theta) = \_\_\_\cos\theta$	$\cos\left(-\frac{\pi}{4}\right) =$	*argument* is ________________.
	$\sin(-\theta) = \_\_\_\sin\theta$		
	$\tan(-\theta) = \_\_\_\tan\theta$	$\csc\left(\frac{\pi}{6}\right) =$	
Pythagorean	$\sin^2\theta + \cos^2\theta = 1$	$\sin^2\frac{11\pi}{6} + \cos^2\frac{11\pi}{6} =$	you see trig functions __________.
	$\_\_\_\_\theta + \_\_\_\_\_ = \_\_\_\_\theta$		
	$\_\_\_\_\_ + \_\_\_\_\theta = \_\_\_\_\theta$		
Sum & Diff.	$\sin(a \pm b) = \_\_\_\_a\ \_\_\_\_b \pm \_\_\_\_a\ \_\_\_\_b$	$\sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right) =$	argument contains a ______/______ *OR* multiples of $15°$ or $\frac{\pi}{12}$
	$\cos(a \pm b) = \_\_\_\_a\ \_\_\_\_b \mp \_\_\_\_a\ \_\_\_\_b$		
	$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$		
Double Angle	$\sin 2\theta = 2\_\_\_\_\_\_\_\_\_\_\_\_$	$\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12} =$	argument contains ______ *OR* you recognize a ________ of the identity
	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $= 1 - 2\sin^2\theta$ $= 2\cos^2\theta - 1$		
	$\tan 2\theta = \frac{2\tan\theta}{1 - \_\_\_\_\_^2\theta}$		

TOPIC: INTRODUCTION TO TRIG IDENTITIES

Simplifying Trig Expressions

◆ You'll need to use ALL trig identities to *fully* simplify expressions.

EXAMPLE Simplify the expression.

(A)

$\tan(-\theta) \cdot \csc\theta$

(B)

$$\frac{\sin^2\theta}{1+\cos\theta}$$

HOW TO: Fully Simplify Trig Expressions

Trig expressions are fully simplified when…

- ▢ all arguments are ____________
- ▢ expression contains NO _____________
- ▢ expression contains as few trig fcns as possible

Strategies:

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
- ◆ Break down in terms of ______ & ______
- ◆ If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- ◆ Factor

Recall **Fundamental Trig Identities**

Reciprocal & Quotient

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Even/Odd

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

Pythagorean

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

TOPIC: INTRODUCTION TO TRIG IDENTITIES

EXAMPLE Simplify the expression.

(A)

$$\frac{\sin^2\theta - \tan^2\theta}{\sin\theta + \tan\theta}$$

(B)

$$\frac{\cos\theta + \csc\theta}{\cos\theta} + \frac{\sin\theta - \sec\theta}{\sin\theta}$$

HOW TO: Fully Simplify Trig Expressions

Trig expressions are fully simplified when…

- ▢ all arguments are positive
- ▢ expression contains NO fractions
- ▢ expression contains as few trig fcns as possible

Strategies:

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
- ◆ Break down in terms of sin & cos
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$\sin(-\theta) = -\sin\theta$	$\sin^2\theta + \cos^2\theta = 1$
$\cos(-\theta) = \cos\theta$	$\tan^2\theta + 1 = \sec^2\theta$
$\tan(-\theta) = -\tan\theta$	$1 + \cot^2\theta = \csc^2\theta$

TOPIC: INTRODUCTION TO TRIG IDENTITIES

PRACTICE Simplify the expression.

(A)

$$\tan^2\theta - \sec^2\theta + 1$$

(B)

$$\frac{\tan(-\theta)}{\sec(-\theta)}$$

(C)

$$\left(\frac{\tan^2\theta}{\sin^2\theta} - 1\right)\csc^2\theta\cos^2(-\theta)$$

HOW TO: Fully Simplify Trig Expressions

Trig expressions are fully simplified when…

- ▢ all arguments are positive
- ▢ expression contains NO fractions
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Strategies:

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
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Recall: Fundamental Trig Identities

Reciprocal & Quotient

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TOPIC: INTRODUCTION TO TRIG IDENTITIES

Verifying Trig Equations as Identities

- To verify that an equation is true, simplify ONE or BOTH sides with the goal of making them __________.
  - ALWAYS start with the more ______________ side first!

EXAMPLE Verify the identity.

(A)

$$\frac{\sin\theta\cos\theta}{1-\cos^2\theta} = \frac{1}{\tan\theta}$$

(B)

$$\frac{\sec^2\theta - \tan^2\theta}{\cos(-\theta)+1} = \frac{1-\cos\theta}{\sin^2\theta}$$

STRATEGIES: Simplifying Trig Expressions

- Constantly scan for identities
- Add fractions using a common denominator
- Break down in terms of sin & cos
- If $1 \pm \mathrm{trig}(\theta)$, multiply top & bottom by $1 \mp \mathrm{trig}(\theta)$
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Recall **Fundamental Trig Identities**

Reciprocal & Quotient

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TOPIC: INTRODUCTION TO TRIG IDENTITIES

EXAMPLE Verify the identity by working with one side.

$$\frac{1-\sin\theta}{\cos\theta}-\frac{\cos\theta}{1+\sin\theta}=0$$

STRATEGIES: Simplifying Trig Expressions

- Constantly scan for identities
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- Factor

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Reciprocal & Quotient

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TOPIC: INTRODUCTION TO TRIG IDENTITIES

EXAMPLE Verify the identity by working with both sides.

$$\sec\theta\,(1-\sin^2\theta)=\frac{(1+\tan^2\theta)\cot^2\theta}{\csc^2\theta\sec\theta}$$

STRATEGIES: Simplifying Trig Expressions

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TOPIC: INTRODUCTION TO TRIG IDENTITIES

PRACTICE Identify the most helpful first step in verifying the identity.

(A)

$$\left(\frac{\tan^2\theta}{\sin^2\theta} - 1\right) = \sec^2\theta\,\sin^2(-\theta)$$

(B)

$$\sec^3\theta = \sec\theta + \frac{\tan^2\theta}{\cos\theta}$$

STRATEGIES: Simplifying Trig Expressions

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TOPIC: SOLVING TRIG EQUATIONS USING IDENTITIES

Solve Trig Equations Using Identity Substitutions

- Recall: Solve trig equations by finding θ that makes the equation true.
  - When given eqns with *multiple* trig fcns, use _______________ to rewrite in terms of *one* trig fcn, then solve.

Recall: Linear Trig Equations	New: Other Trig Equations
$\tan\theta = 1$ ↓ $\theta = \frac{\pi}{4} + \pi n$	$\frac{\sec^2\theta - 1}{\tan\theta} = 1$ ↓ $\frac{\qquad}{\tan\theta} = 1$ ↓ $\tan\theta = 1$ ↓ $\theta = \frac{\pi}{4} + \pi n$ Substitute using ___________________ Identity

EXAMPLE Find all solutions to the equation.

$$\frac{\sin 2\theta}{\cos(-\theta)} = 1$$

Rewrite **[TOP | BOTTOM]** using _________________ Identity

Rewrite **[TOP | BOTTOM]** using _________________ Identity

Solve _____________ trig equation

TOPIC: SOLVING TRIG EQUATIONS USING IDENTITIES

PRACTICE Find all solutions to the equation.

$$(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = -\frac{1}{2}$$

PRACTICE Find all solutions to the equation where $0 \leq \theta \leq 2\pi$.

$$\sin\theta\cos(2\theta) - \sin(2\theta)\cos\theta = \frac{\sqrt{2}}{2}$$

CHAPTER RESOURCE: THE UNIT CIRCLE