

TOPIC: INDEFINITE INTEGRALS

Introduction to Indefinite Integrals

◆ Recall: If $f(x)$ is the derivative of a function $F(x)$, then $F(x)$ is an antiderivative of $f(x)$.

► Finding the indefinite integral of a function is the _____ as finding the antiderivative.

NewIndefinite Integrals

$$\int \underset{\substack{\text{integrand} \\ \downarrow \\ \text{_____ of integration}}}{f(x)} d\underset{\substack{\downarrow \\ \text{_____ of integration}}}{x} = \underset{\substack{\text{antiderivative} \\ \downarrow \\ \text{_____ of integration}}}{F(x)} + \underset{\substack{\downarrow \\ \text{_____ of integration}}}{C}$$

EXAMPLE

Find the indefinite integral of the following functions.

(A) $\int 0 \, dx$

New

 $\int 0 \, dx = \underline{\hspace{2cm}}$

(B) $\int 3 \, dx$

New

 $\int k \, dx = \underline{\hspace{2cm}}$

(C) $\int 3x^2 \, dx$

◆ Just like with antiderivatives, you can check your answer to an indefinite integral by taking the derivative of it.

TOPIC: INDEFINITE INTEGRALS

PRACTICE

Find the following indefinite integral.

$$\int -300 \, dx$$

PRACTICE

Find the following indefinite integral.

$$\int 4x^3 \, dx$$

TOPIC: INDEFINITE INTEGRALS

PRACTICE

Find the following indefinite integral.

$$\int 100x^{99} dx$$

TOPIC: INDEFINITE INTEGRALS

Power Rule for Indefinite Integrals

◆ Recall: To find the **derivative** of a power fcn $f(x) = x^n$, *multiply* by original exponent, then *decrease* exponent by 1.

► To find the **integral** of a power fcn $f(x) = x^n$, _____ exponent by 1, then _____ by the new exponent.

Recall

$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

Power Rule for Derivatives

RULES OF INTEGRATION

| Name | Rule | Example |
|-------|--|---|
| Power | $\int x^n dx = \frac{x}{n} + C, \quad n \neq -1$ | $\int x^6 dx = \frac{x}{6} + C = \frac{x}{6} + C$ |

EXAMPLE

Find the indefinite integral of $g(t) = t^4$.

TOPIC: INDEFINITE INTEGRALS

PRACTICE

Find $h(x)$ by evaluating the following indefinite integral.

$$h(x) = \int x^8 dx$$

PRACTICE

Find $h(x)$ by evaluating the following indefinite integral.

$$h(x) = \int x^{100} dx$$

TOPIC: INDEFINITE INTEGRALS

PRACTICE

Find $h(x)$ by evaluating the following indefinite integral.

$$h(x) = \int 25x^4 \, dx$$

TOPIC: INDEFINITE INTEGRALS

Additional Rules for Indefinite Integrals

- ◆ The same rules that apply to the sum, difference, or constant multiple of a derivative also apply to integrals.

| RULES OF INTEGRATION | | |
|----------------------|--|---|
| Name | Rule | Example |
| Sum & Difference | $\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$ | $\int (x + 6) dx = \int \underline{\hspace{1cm}} dx + \int \underline{\hspace{1cm}} dx =$ |
| Constant Multiple | $\int [k \cdot f(x)] dx = \underline{\hspace{1cm}} \cdot \int f(x) dx$ | $\int 5x^3 dx = \underline{\hspace{1cm}} \cdot \int \underline{\hspace{1cm}} dx =$ |

- ◆ Use *multiple* rules together to find integrals of more complicated functions.

EXAMPLE

Find the indefinite integral of $f(x) = x^2 - 3x + 6$.

TOPIC: INDEFINITE INTEGRALS

PRACTICE

Find $f(x)$ by evaluating the following indefinite integral.

$$f(x) = \int (100x^2 - 35x - \frac{13}{2})dx$$

PRACTICE

Find $f(x)$ by evaluating the following indefinite integral.

$$f(x) = \int (8x^7 + 10x - 20)dx$$