Graphs of Common Functions

- There are several graphs that may __________ show up in this course.

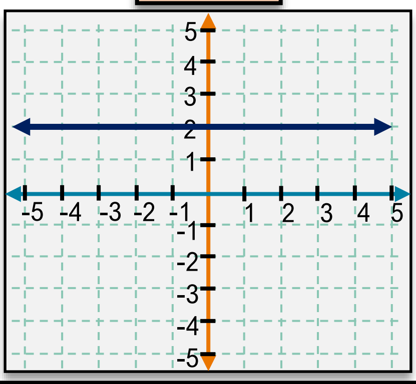
Graphs of Common Functions

Constant Function
(y = same value, does NOT depend on x)

$f(x) = c$

Domain: ________

Range: ________

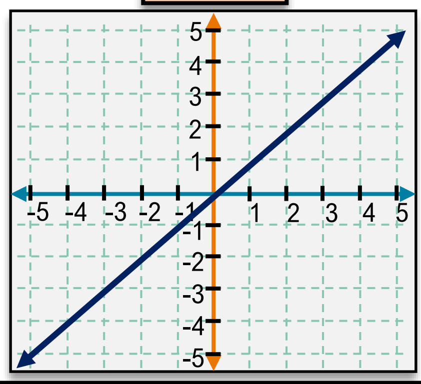


Identity Function

$f(x) = x$

Domain: ________

Range: ________



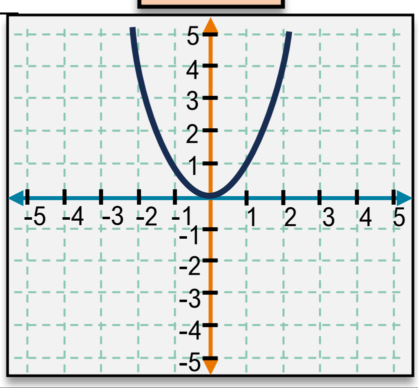
Square Function
(Output = square of input)

$f(x) = x^2$

- **Shape is a** __________

Domain: ________

Range: ________

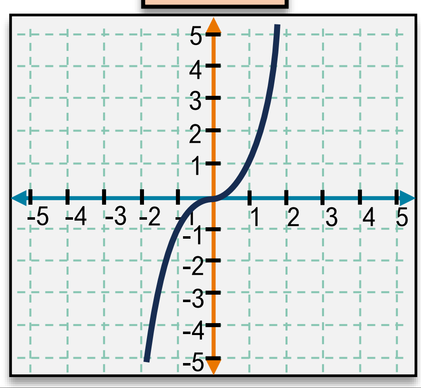


Cube Function
(Output = cube of input)

$f(x) = x^3$

Domain: ________

Range: ________



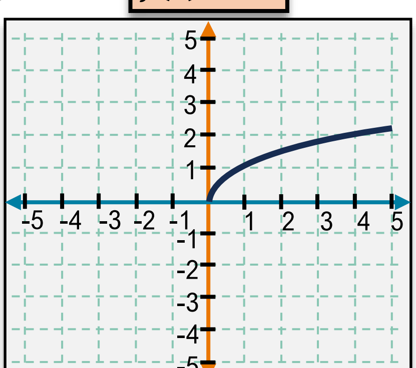
Square Root Function
(Output is the square root of input)

- x **[CAN | CANT]** be negative

$f(x) = \sqrt{x}$

Domain: ________

Range: ________



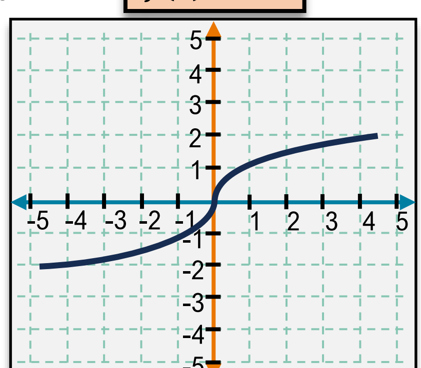
Cube Root Function
(Output is the cube of input)

- x **[CAN | CANT]** be negative

$f(x) = \sqrt[3]{x}$

Domain: ________

Range: ________

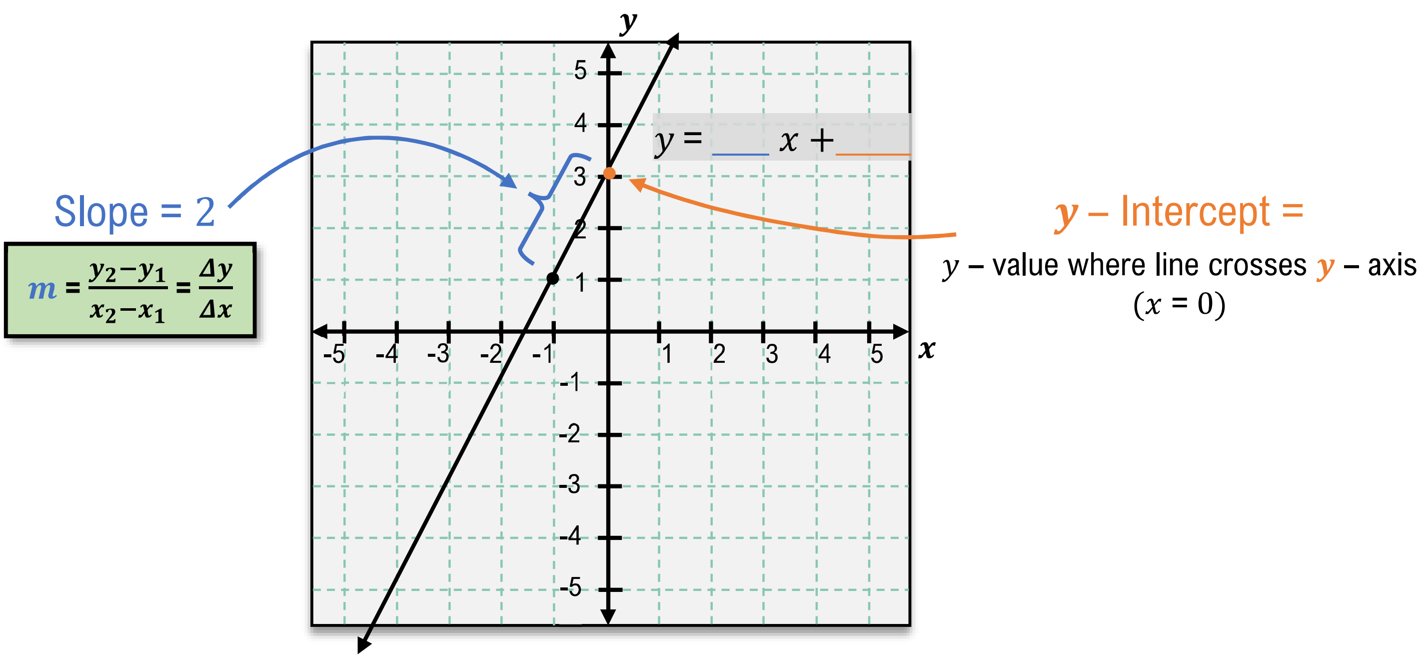


Slope – Intercept Form

● We can write the equation of a line using its ____________ & _________________.

$y = \_\_x + \_\_$

(*Slope – Intercept Form*)

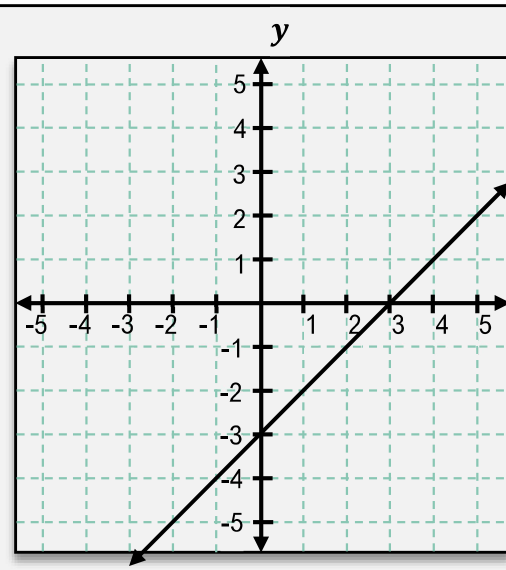


EXAMPLE: In the graph below, identify the y – intercept & slope.

Write the equation in slope-intercept form.

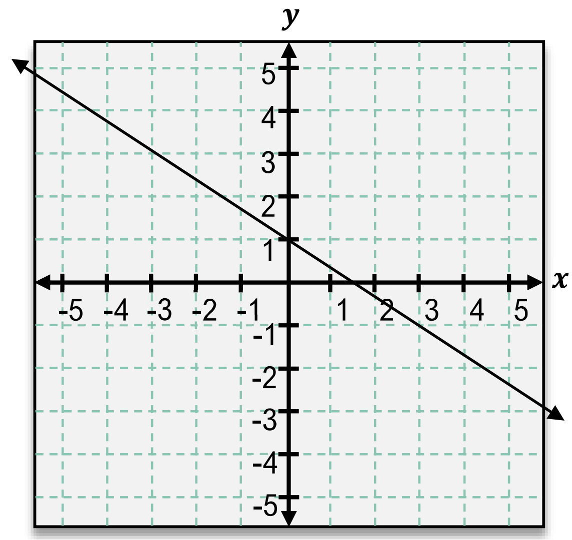
b = _______

m = _______



PRACTICE: In the graph shown, identify the y – intercept & slope. Write the equation of this line in Slope-Intercept form.

$$y = mx + b$$



TOPIC: QUADRATIC FUNCTIONS

Properties of a Parabola

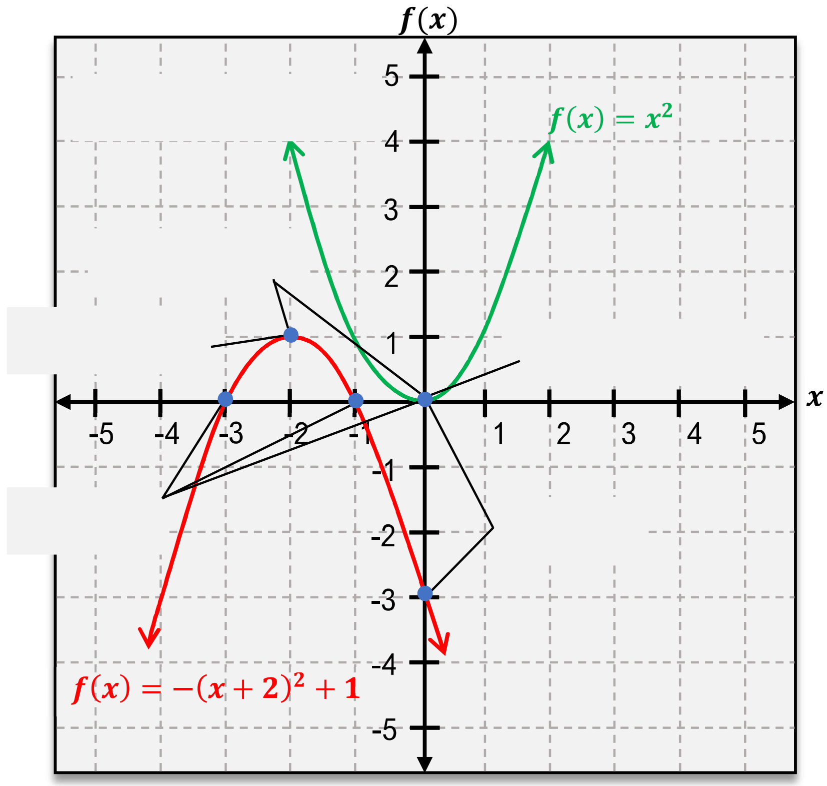
- A **quadratic function** is a polynomial of degree ___ in the **standard form**: $f(x) = ax^2 + bx + c$

$f(x) = x^2$	$f(x) = 2x^2 + 3x - 7$	$f(x) = \frac{2}{3}x^2 + 1$

  - $\boldsymbol{a, b, c}$ can be any real number as long as $a \neq$____.
  - Recall: The square function is a _______________, as ***all*** quadratic functions will be.

$f(x) = x^2$
Vertex: _______ [MIN|MAX]
x-intercept(s): ___________
y-intercept: _________
Axis of Symmetry: ________
Domain: *always* _________
Range when [MIN], __________: _________

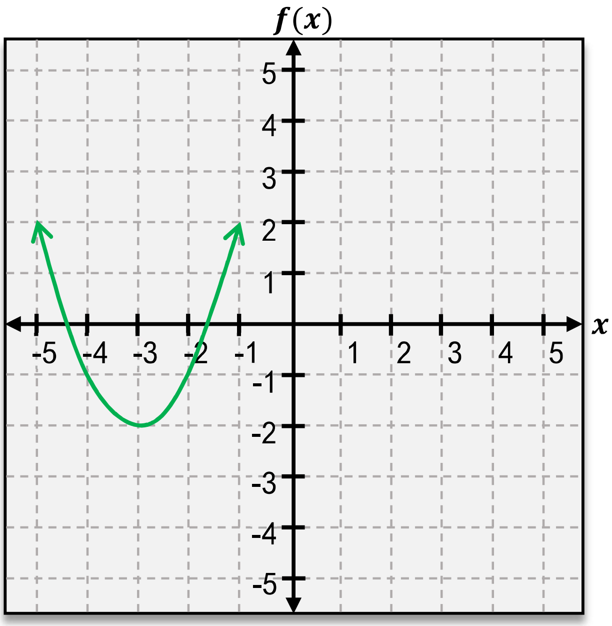
$f(x) = -(x+2)^2 + 1$
Vertex: _______ [MIN|MAX]
x-intercept(s): ___________
y-intercept: _________
Axis of Symmetry: ________
Domain: _______
Range when [MAX], __________: _________
Increasing? ___________
Decreasing? ___________



- Quadratic functions are commonly written in **vertex form**, which will help us graph with ease.

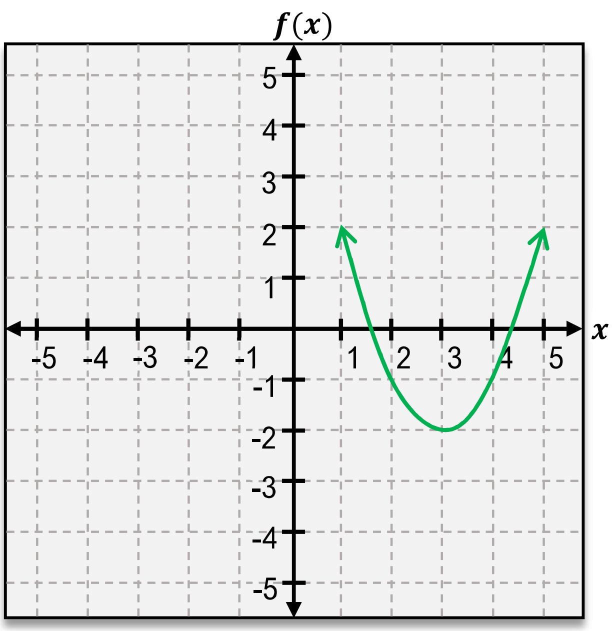
TOPIC: QUADRATIC FUNCTIONS

PRACTICE: Identify the ordered pair of the vertex of the parabola. State whether it is a minimum or maximum.



Vertex: ________ [MIN|MAX]

PRACTICE: Where is the axis of symmetry located on the given parabola?



Axis of Symmetry: ________

TOPIC: UNDERSTANDING POLYNOMIAL FUNCTIONS

Intro to Polynomial Functions

● You will need to know how recognize polynomial functions & their graphs.

- Recall: Polynomials have *only* positive whole number exponents (*no negatives, no fractions*)
- Standard form: Like terms combined & in descending order of power $f(x) \;= a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$

Degree n (highest exponent)

$$\_\_\_\_\_\_= 6x^3 + 3x^2 + 5x + 4$$

Leading Coefficient   Coefficients   Constant

EXAMPLE: Determine if each function is a polynomial function. If so, put in standard form. State degree & leading coeff.

(A)

$f(x) = -x^2 + 5x^3 - 6x + 4$

Polynomial function? ☐

Degree: _______

Leading Coefficient: _______

(B)

$f(x) = 2x^{\frac{1}{2}} + 3$

Polynomial function? ☐

Degree: _______

Leading Coefficient: _______

(C)

$f(x) = -\frac{2}{3}x^4 + 1 + 3$

Polynomial function? ☐

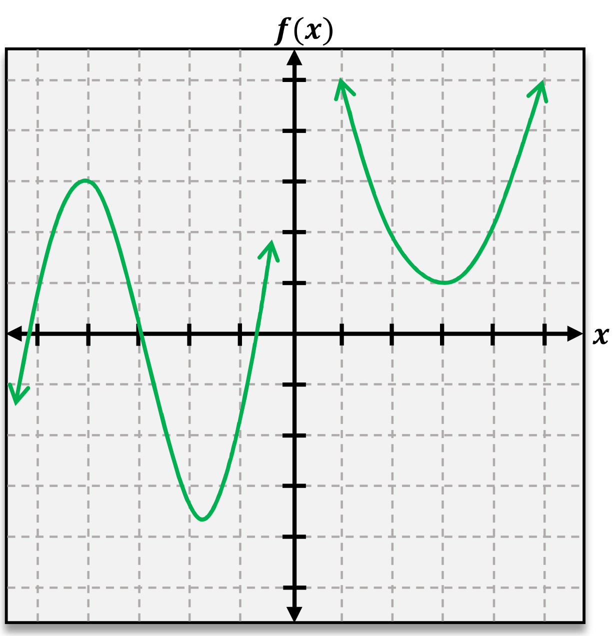
Degree: _______

Leading Coefficient: _______

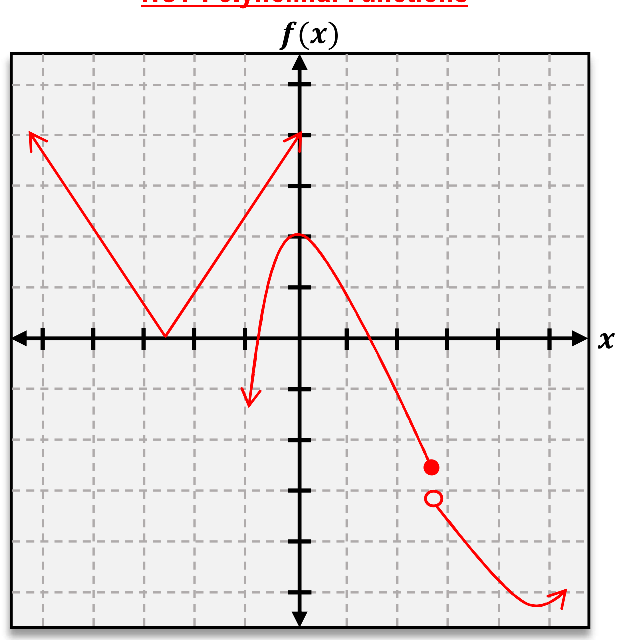
Graphs of Polynomial Functions

● Graphs of polynomial functions are ____________ and _______________ (*no corners, no breaks*)

Polynomial Functions



NOT Polynomial Functions



- Domain: *always* ___________

TOPIC: UNDERSTANDING POLYNOMIAL FUNCTIONS

PRACTICE**:** Determine if the given function is a polynomial function. If so, write in standard form, then state the degree and leading coefficient.

$$f(x) = 4x^3 + \frac{1}{2}x^{-1} - 2x + 1$$

Polynomial function? ☐

Degree**:** _______

Leading Coefficient**:** _______

PRACTICE**:** Determine if the given function is a polynomial function. If so, write in standard form, then state the degree and leading coefficient.

$$f(x) = 2 + x$$

Polynomial function? ☐

Degree**:** _______

Leading Coefficient**:** _______

PRACTICE**:** Determine if the given function is a polynomial function. If so, write in standard form, then state the degree and leading coefficient.

$$f(x) = 3x^2 + 5x + 2$$

Polynomial function? ☐

Degree**:** _______

Leading Coefficient**:** _______

Introduction to Rational Functions

- A **rational function** has a ______________ in the numerator & denominator: $f(x) = \frac{x^2+4x+1}{3x+2} = \frac{p(x)}{q(x)}$
  - Recall: The denominator of a fraction ***CANNOT*** be ________.

Rational ***Equation***	Rational ***Function***
$12 = \frac{1}{x-1}$, $\underbrace{x-1}_{\neq 0}$	$f(x) = \frac{1}{x-1}$, $\underbrace{x-1}_{\neq 0}$
Restriction: $x \neq$ ____	Domain: $\{x \mid x \neq$ ____$\}$

- To determine domain, set denominator = 0 & solve for x. Domain is any real #, *EXCEPT* what makes denom $= 0$.
- To write a rational function in **lowest terms**, factor top & bottom, then __________ any common factors.
  - Always find the domain *BEFORE* writing in lowest terms.

<u>EXAMPLE</u>: Find the domain of the rational function. Then, write the function in lowest terms.

(A)

$f(x) = \frac{3}{3x+12}$

$\{x \mid x \neq$ ________$\}$

$f(x) = \frac{3}{3x+12}$

(B)

$f(x) = \frac{x+5}{x^2-25}$

$\{x \mid x \neq$ ________$\}$

$f(x) = \frac{x+5}{x^2-25}$

PRACTICE: Find the domain of the rational function. Then, write it in lowest terms.

$f(x) = \frac{x^2+9}{x-3}$ $\{x | x \neq$ ________$\}$

<u>PRACTICE</u>**:** Find the domain of the rational function. Then, write it in lowest terms.

$f(x) = \frac{6x^5}{2x^2-8}$ $\{x|x \neq \_\_\_\_\_\_\_\_\}$

TOPIC: INTRODUCTION TO EXPONENTIAL FUNCTIONS

Exponential Functions

◆ **Polynomial** functions have a variable base with a number exponent; **exponential** functions have the opposite!

- Exponential functions have a:
  - **Base** that is _________, __________, & ___ 1.
  - **Exponent** (power) that contains a __________.

Recall **Polynomial Function**	New **Exponential Function**
$f(x) = x^2$	$f(x) = 2^x$ (Power: x; Base: 2)

EXAMPLE Determine if each function is an exponential function.

(**A**) $f(x) = \left(\frac{2}{3}\right)^x$

Exponential function? ☐

Power: _______

Base: ________

(**B**) $f(y) = 1^y$

Exponential function? ☐

Power: _______

Base: ________

(**C**) $f(x) = 10^{x+1}$

Exponential function? ☐

Power: _______

Base: ________

◆ You will be asked to evaluate exponential functions for specified values of x.

- For exponents that cannot easily be done by hand, type (BASE) [ ] (POWER) into a calculator.

EXAMPLE Evaluate the exponential function $f(x) = 2^x$ for each given value of x.

(**A**) $x = 4$

(**B**) $x = -3$

(**C**) $x = 3.14$

(**D**) $x = 12$

TOPIC: INTRODUCTION TO EXPONENTIAL FUNCTIONS

PRACTICE Determine if each function is an exponential function. If so, identify the power & base, then evaluate for $x = 4$.

(***A***)

$f(x) = (-2)^x$

Exponential function? ☐

Power: _______

Base: ________

$\boldsymbol{f(4)} =$_________

(***B***)

$f(x) = 3(1.5)^x$

Exponential function? ☐

Power: _______

Base: ________

$\boldsymbol{f(4)} =$_________

(***C***)

$f(x) = \left(\frac{1}{2}\right)^x$

Exponential function? ☐

Power: _______

Base: ________

$\boldsymbol{f(4)} =$_________

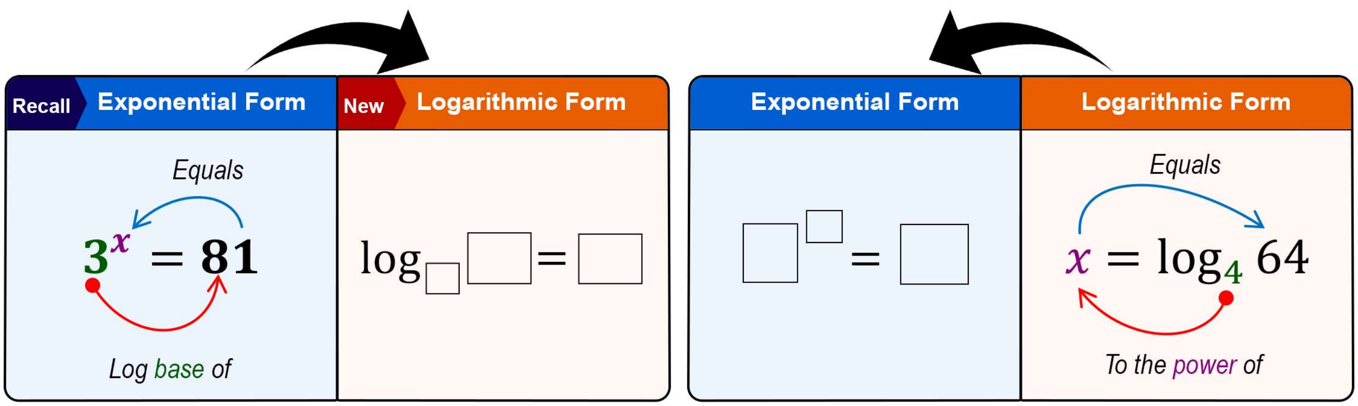
TOPIC: INTRODUCTION TO LOGARITHMS

Logarithms Introduction

- The _______________ (inverse) operation of an exponential is taking the ***logarithm*** (log).
  - Logs and exponentials with the same **base** ________ each other.
  - A **log** gives us the **power** that some **base** must be raised to in order to equal a particular number.

Solving Polynomials	Solving Exponentials		
	(**A**)	(**B**)	
$\mathbf{x^3 = 216}$	$\mathbf{2^x = 8}$	$\mathbf{2^x = 216}$	*(Exponential Form)*
$\sqrt[3]{\mathbf{x^3}} = \sqrt[3]{\mathbf{216}}$	$\mathbf{2 \times 2 \times 2 = 8}$		
$\mathbf{x = \sqrt[3]{216}}$	$\mathbf{x = 3}$	$\mathbf{x = \log}$	*(Logarithmic Form)* *"log base 2 of 216"*

- You will need to convert expressions between **exponential form** and **logarithmic form**.



EXAMPLE Write each log in exponential form & each exponential in log form.

(**A**)	(**B**)	(**C**)
$x = \log_5 800$	$\log_2 16 = 4$	$10^x = 4500$

- $\log_{10}$, known as the __________ log, can be written as just _____ and has its own calculator button: LOG

TOPIC: GRAPHING LOGARITHMIC FUNCTIONS

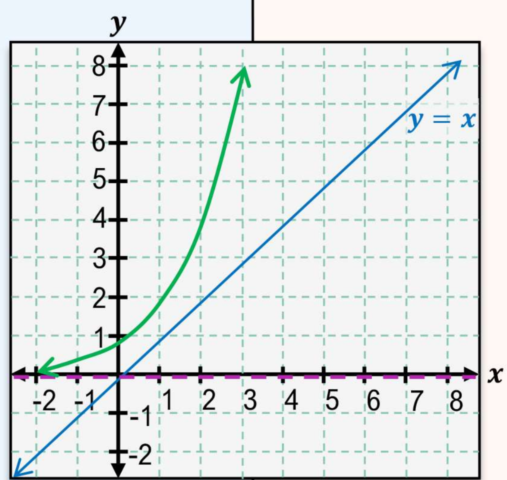
Graphs of Logarithmic Functions

◆ We can graph a logarithmic function using the fact that it is the ____________ of an exponential function.

- $f(x) = \log_b x$ can be graphed by ______________ the graph of its inverse function, $y = b^x$ over ________.

Recall — **Exponential Functions** | **New** — **Logarithmic Functions**

x	$f(x) = 2^x$
-2	¼
-1	½
0	1
1	2
2	4
3	8



x	$f(x) = \log_2 x$
¼	
½	
1	
2	
4	
8	

Domain: *always* $\mathbb{R}$ — Range: depends on asymp.; $(0, \infty)$

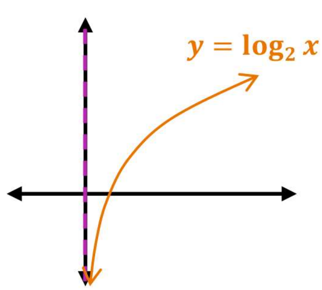
Horizontal Asymptote at $y = 0$ - - - → ___________ Asymptote at ______

Domain: depends on _________; _______

Range: *always* ________

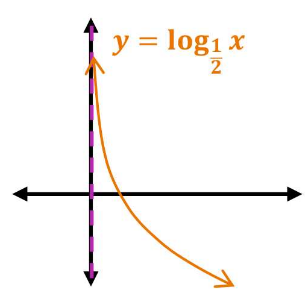
◆ Just like its inverse, the direction of the graph of $f(x) = \log_b x$ depends on ___.

$b > 1$



◆ Graph **[INCREASES | DECREASES]**

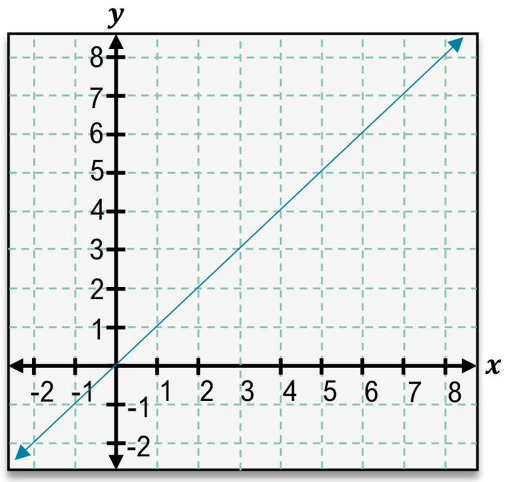
$0 < b < 1$



◆ Graph **[INCREASES | DECREASES]**

TOPIC: GRAPHING LOGARITHMIC FUNCTIONS

EXAMPLE: Graph $f(x) = 3^x$ and $g(x) = \log_3 x$ on the graph below. Determine the domain and range of each.



x	$f(x) = 3^x$
-2	
-1	
0	
1	
2	

Domain: __________

Range: _________

x	$g(x) = \log_3 x$

Domain: __________

Range: _________