TOPIC: AVERAGE VALUE OF A FUNCTION

Average Value of a Function

◆ Recall: Taking the limit as $n \to \infty$ of a Riemann sum gives the *true* area under the curve: the definite integral.

► We can use a definite integral to find the average value of a function $f(x)$ from $[a, b]$.

New **Average Value of a Function**

$$\text{Avg value} \approx \frac{f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)}{\quad}$$

$$\text{Avg value} \approx \frac{1}{\quad} \sum_{k=1} \_\_\_\_\_$$

$$\text{Avg value} \approx \frac{\quad}{\quad} \sum_{k=1}^{n} f(x_k^*)$$

$$\text{Avg value} \approx \frac{1}{\quad} \sum_{k=1}^{n} f(x_k^*) \_\_\_$$

$$\text{Avg value} = \frac{1}{b-a} \cdot \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

Recall

$$\Delta x = \frac{b-a}{n}$$

Recall

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

$$f_{avg} = \frac{1}{b-a} \int_a^b f(x)\,dx$$

EXAMPLE Find the average value of $f(x) = x + 2$ on $[0,4]$.

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PRACTICE Find the average value of the function on the interval $[2,5]$.

$$F(x) = x^3 - \frac{2}{\sqrt{x}}$$

PRACTICE Find the average value of the function on the interval $[4,9]$.

$$F(x) = \sqrt{x} + 6$$

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PRACTICE Find the average value of the function on the interval $[0,1]$.

$$G(x) = \frac{2}{x^2 + 1}$$

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EXAMPLE Consider the function $G(x) = 3x^2 - 2x + 10$.

$(\boldsymbol{A})$ Find the average value of $G(x)$ on the interval $[2,4]$.

$(\boldsymbol{B})$ Sketch the graph of $G(x)$ on this interval and indicate the average value on the graph.