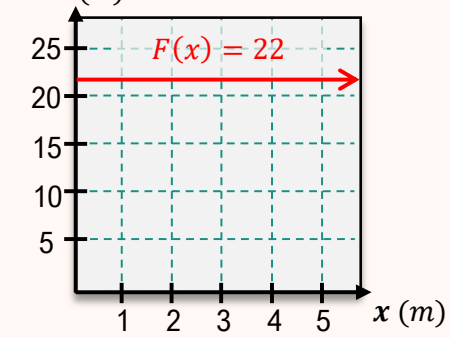
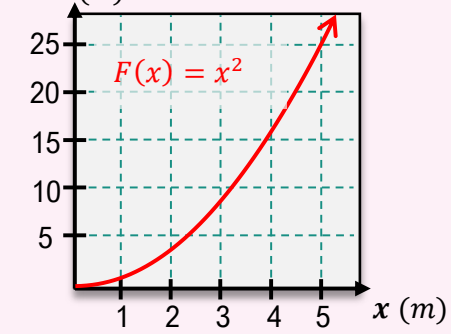
TOPIC: WORK

Introduction to Work

◆ Work is done when a force is applied on an object over a distance.

► Work is the area under the force curve. Units: Newton-meters (Joules) or foot-pounds.

EXAMPLE Find the work done on the object by the force (in Newtons) $F(x)$ from $x = 1\ m$ to $x = 4\ m$.

Work Done by *Constant* Force	Work Done by *Variable* Force
Force (N)  $Work =$ $\times$ $W =$ _____ $\times$ _____ $=$	**Force** (N)  $Work = \int_{\_}^{\_} \_\_\_\ dx$ $W = \int_{\_}^{\_} \_\_\_\ dx =$

TOPIC: WORK

PRACTICE

How much work is required to push a chair across the floor with a force of $F = 8\ lb$ from $x = 6\ ft$ to $x = 9\ ft$ along the x-axis?

PRACTICE

How much work is done by a person lifting a $10\ lb$ bucket $4\ ft$ off the ground?

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PRACTICE

How much work is required to move an object with a force of $F(x) = 4x^3\ N$ acting along the x-axis from $x = 0\ m$ to $x = 2\ m$?

PRACTICE

Compute the work done by a force $F = \frac{3}{x^2}\ N$ from $x = 2\ m$ to $x = 6\ m$.

TOPIC: WORK

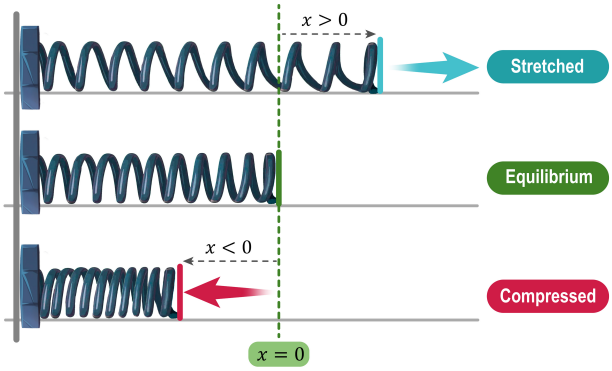
Work Done on a Spring (Hooke's Law)

◆ Recall: Work done by a force acting on an object from $x = a$ to $x = b$ is

Recall

$$W = \int_a^b F(x)\,dx$$

► The force required to compress/stretch a spring depends on its stiffness, given by the spring constant k.



EXAMPLE Given a spring constant $k = 3\ N/m$, how much work is done to stretch the spring from $4\ m$ to $6\ m$?

New **Work Done on a Spring**

Recall

$$W = \int_a^b F(x)\,dx$$

$F(x) =$ _____

$$W = \int_{\_}^{\overline{\ }} \_\_\_\_\_\ dx$$

HOW TO: Find Work Done on a Spring

1) Plug k into $F(x) = kx$

If not given, find:

$$k = \frac{F}{x} = \frac{force\ applied}{distance}$$

2) Determine bounds $[a, b]$

a is _____ dist. from equilibrium

b is _____ dist. from equilibrium

3) Find $W = \int_a^b F(x)dx$

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PRACTICE

A spring requires $12\ J$ of work to stretch the spring from $1.1\ m$ to $1.4\ m$ past its equilibrium. What is the spring constant?

PRACTICE

A spring requires a force of $8\ N$ to stretch the spring to $5\ cm$ past its equilibrium point. How much work would it take to stretch the spring from $2\ cm$ to $9\ cm$ past equilibrium?

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PRACTICE

Suppose a force of $10\ N$ is required to stretch a spring $0.5\ m$ from its equilibrium position. How much work is required to compress the spring $0.2\ m$ from its equilibrium position?

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EXAMPLE A spring has a natural length of $8\ in$. It takes $24\ lb$ to stretch the spring $12\ in$.

$(\boldsymbol{A})$ Find the spring constant k.

$(\boldsymbol{B})$ How much work would it take to stretch the spring from $12\ in$ to $16\ in$?

TOPIC: WORK

Lifting Problems

◆ Recall: Work done by a force acting on an object from $x = a$ to $x = b$ is

Recall

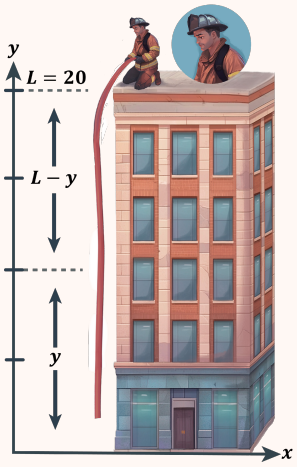
$$W = \int_a^b F(x)\,dx$$

► To find work needed to lift an object, the force is equivalent to its _____.

EXAMPLE A firefighter holds a $20\ m$ rope with density of $3\ kg/m$ from the top of a $20\ m$ building. Find work required to lift the rope $h = 10\ m$ at a constant speed.

New **Lifting Problems**

$$W = \int_0^h \left(\frac{\textit{rope weight}}{\textit{rope length}} * \textit{length}\right) dy$$



Weight per length:

Length function:

$$W = \int_{\_\_}^{\overline{\quad}} \_\_\_\_\_ * \_\_\_\_\_ dy$$

HOW TO: Solve Lifting Problems

1) Sketch picture of lifting problem

2) Find **weight per length** of rope
if in lb/ft, plug in directly
if in kg/m, multiply by _____ first

3) Find **length function:** _____$- y$

4) Determine bounds:
lower = _____
upper = height rope is **lifted** (h)

5) Integrate w.r.t. y

EXAMPLE If there is an $8\ kg$ bucket attached at the end of the rope above, what is the total work required to lift the rope and bucket $h = 10\ m$?

Work to lift rope $10\ m$:

Work to lift bucket $10\ m$:

Total work:

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PRACTICE

A $120\ m$ chain hangs freely from the side of a building. The chain weighs $15\ kg/m$. How much work is done to pull $80\ m$ of the chain to the top of the building?

PRACTICE

Find the work done by fully winding up a cable of length $30\ ft$ and weight-density $2\ lb/ft$.

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PRACTICE

A $40\ m$ rope hangs freely over a ledge. The density of the rope is $12\ kg/m$. If a $5\ kg$ bucket is attached the end of the rope, how much work is done to pull the rope and the bucket to the ledge?

PRACTICE

A $60\ ft$ cable is attached to a cylinder that is attached to a winch. If the cable weighs $300\ lbs$, how much work is needed to wind $20\ ft$ of the cable onto the cylinder using the winch?
Hint: Divide cable weight by cable length to get density.

TOPIC: WORK

Pumping Liquids

◆ Recall: Work done by lifting an object relies on its weight and the distance it is lifted.

Recall

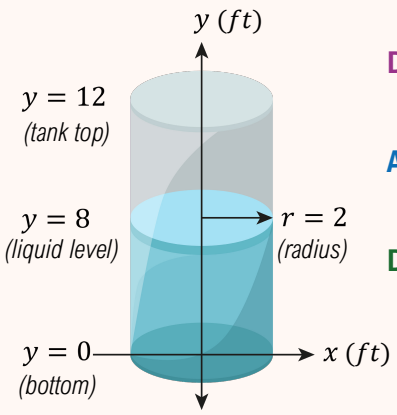
$$W = \int_a^b F(x)\,dx$$

EXAMPLE

A cylindrical tank of radius $2\ ft$ and a height of $12\ ft$ is $8\ ft$ full of oil that weighs $57\ lb/ft^3$. Set up the integral to find the work required to pump the oil to the top of the tank.

New **Pumping Liquids**

$$W = \int_a^b (\textit{density} * \textit{area} * \textit{distance})\,dy$$



Density:

Area:

Distance:

$$W = \int_{\_}^{\_} \_\_\_\_\_\ dy$$

HOW TO: Solve Pumping Problems

1) Sketch picture of tank/liquid

2) Find **density** of the liquid:
if in lb/ft^3, plug in directly
if in kg/m^3, multiply by _____ first

3) Find **area** of cross-section (fcn w.r.t. y)

4) Find **distance** fcn: _____ $- y$

5) Determine bounds:
lower = bottom of *tank* (usually _____)
upper = top layer of *liquid*

6) Integrate w.r.t. y

◆ The cross-sectional area function will change depending on the shape of the tank.

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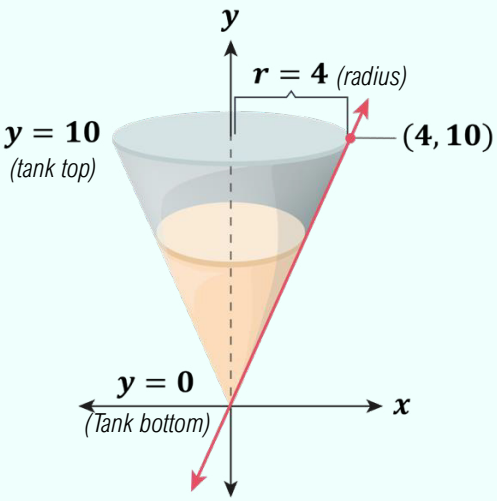
EXAMPLE A tank is in the shape of an inverted cone and is $10\ m$ tall and has a radius of $4\ m$. If the tank is full of gasoline, how much work is required to empty the tank? (Gasoline has a weight-density of $6{,}670\ N/m^3$).



Recall

$$W = \int_a^b (\textit{density} * \textit{area} * \textit{distance})\, dy$$

HOW TO: Solve Pumping Problems

1) Sketch picture of tank/liquid

2) Find **density** of the liquid:
if in lb/ft^3, plug in directly
if in kg/m^3, multiply by 9.8 first

3) Find **area** of cross-section (fcn w.r.t. y)

4) Find **distance** fcn: $L - y$

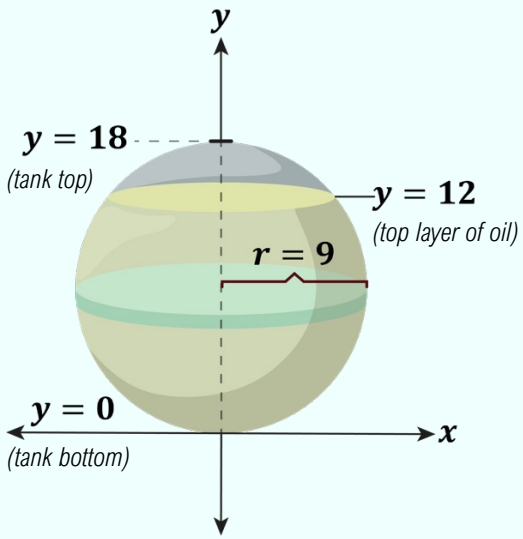
5) Determine bounds:
lower = bottom of *tank* (usually 0)
upper = top layer of *liquid*

6) Integrate w.r.t. y

TOPIC: WORK

EXAMPLE

A spherical tank with an inner radius of $9\ ft$ is two-thirds full of oil that weighs $50\ lb/ft^3$. Find the work required to pump oil out through a hole in the top of the tank.



Recall

$$W = \int_a^b (\textit{density} * \textit{area} * \textit{distance})\, dy$$

HOW TO: Solve Pumping Problems

1) Sketch picture of tank/liquid

2) Find **density** of the liquid:
if in lb/ft^3, plug in directly
if in kg/m^3, multiply by 9.8 first

3) Find **area** of cross-section (fcn w.r.t. y)

4) Find **distance** fcn: $L - y$

5) Determine bounds:
lower = bottom of *tank* (usually 0)
upper = top layer of *liquid*

6) Integrate w.r.t. y

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PRACTICE

A water trough for horses has a triangular cross section with a height of $3\ m$ and horizontal side lengths of $2\ m$. The length of the trough is $12\ m$. How much work is required to pump the water to the top of the trough when it is half full?

PRACTICE

A swimming pool has the shape of a rectangular prism with a base that measures $30\ ft$ by $20\ ft$ and is $5\ ft$ deep. The top of the pool is $1\ ft$ above the surface of the water. How much work is required to pump all the water out? Assume the density of water is $62.4\ lb/ft^3$.

TOPIC: WORK

EXAMPLE

A cylindrical tank of length $12\ ft$ and a radius of $3\ ft$ lay on its side and is full of molasses ($100\ lb/ft^3$). Set up the integral to find work required to empty the tank through a pipe at the top of the tank.