TOPIC: DIFFERENTIABILITY

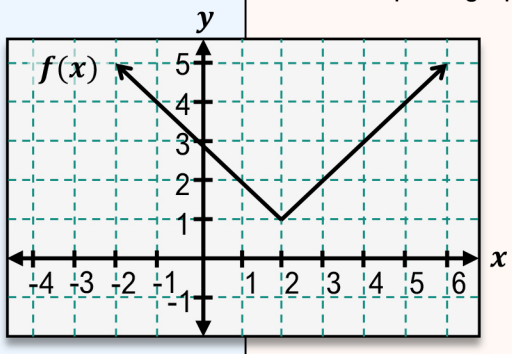
Determine Differentiability Graphically

◆ Recall: $f(x)$ is continuous where you can draw "without picking up your pencil" (no *holes*, *jumps*, *asymptotes*).

- A function is **differentiable** (i.e. *derivative exists*) wherever it's continuous AND _____ (no *sharp corners*).

EXAMPLE Determine if $f(x)$ is $(\boldsymbol{A})$ continuous at $x = 2$ and $(\boldsymbol{B})$ differentiable at $x = 2$.

Recall **Continuity**	New **Differentiability**
"without picking up pencil"	"without picking up pencil ***or drawing sharp corners***"
$f(x)$ is continuous at a: ☐	$f(x)$ is continuous at a: ☐
$f(a)$ exists ☐	$f(a)$ exists ☐
$\lim_{x\to a} f(x)$ exists ☐	$\lim_{x\to a} f(x)$ exists ☐
$\lim_{x\to a} f(x) = f(a)$ ☐	$\lim_{x\to a} f(x) = f(a)$ ☐
	$f'(a)$ exists: ☐

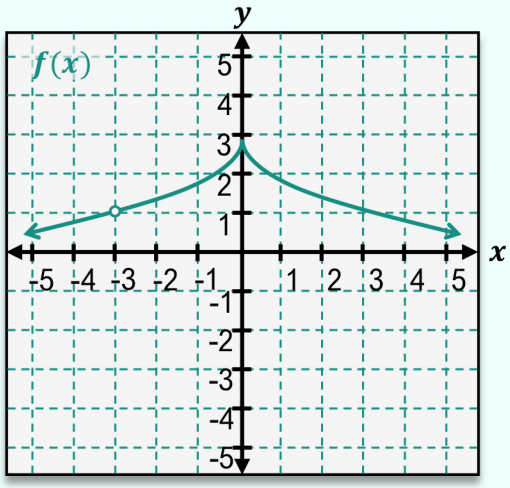


EXAMPLE For each interval or x-val, determine if $f(x)$ is continuous and/or differentiable.

$(\boldsymbol{A})$ $x = -3$ Continuous? ☐ Differentiable? ☐

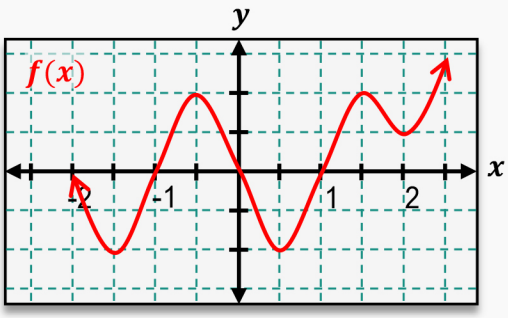
$(\boldsymbol{B})$ $x = 0$ Continuous? ☐ Differentiable? ☐

$(\boldsymbol{C})$ $(0, \infty)$ Continuous? ☐ Differentiable? ☐

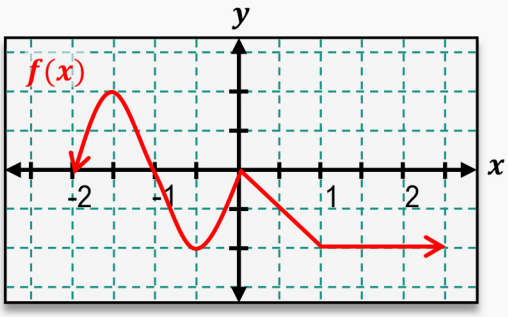


TOPIC: DIFFERENTIABILITY

PRACTICE Determine if the graph of the function $f(x)$ is continuous and/or differentiable at $x = 2$.

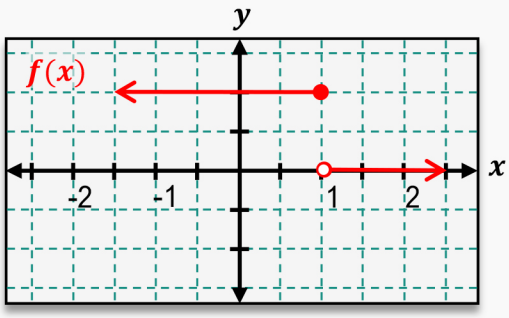


PRACTICE Determine if the graph of the function $f(x)$ is continuous and/or differentiable at $x = 1$.



TOPIC: DIFFERENTIABILITY

PRACTICE Determine if the graph of the function $f(x)$ is continuous and/or differentiable at $x = 1$.



TOPIC: DIFFERENTIABILITY

Determine Differentiability Without a Graph

◆ Recall: A fcn is differentiable (i.e. *derivative exists*) wherever it's **continuous** & **smooth** (no *sharp corners*).

- Polynomials are always differentiable. For piecewise fcns, check $f(x)$ & $f'(x)$ at _____(s) between the pieces.

EXAMPLE Determine if the piecewise function below is $(\boldsymbol{A})$ continuous and/or $(\boldsymbol{B})$ differentiable.

$$f(x) = \begin{cases} x & \text{for } x < 2 \\ x^2 - 2 & \text{for } x \geq 2 \end{cases}$$

Recall

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Continuous: $f(a)_{left} = f(a)_{right}$ ☐

Differentiable: $f'(a)_{left} = f'(a)_{right}$ ☐

TOPIC: DIFFERENTIABILITY

PRACTICE Determine if the function $f(x)$ is continuous and/or differentiable at $x = 3$.

$$f(x) = \begin{cases} x^2 & \text{for } x < 3 \\ 2x + 3 & \text{for } x \geq 3 \end{cases}$$

PRACTICE Determine if the function $f(x)$ is continuous and/or differentiable at $x = 2$.

$$f(x) = \begin{cases} x^3 & \text{for } x < 2 \\ (x-2)^2 & \text{for } x \geq 2 \end{cases}$$

TOPIC: DIFFERENTIABILITY

PRACTICE Determine where the function $f(x)$ is not differentiable.

$$f(x) = \frac{3}{x+2}$$

PRACTICE Determine if the function $f(x)$ is continuous and/or differentiable at $x = 3$.

$$f(x) \begin{cases} x^2 & \text{for } x < 3 \\ \frac{27}{x} & \text{for } x \geq 3 \end{cases}$$