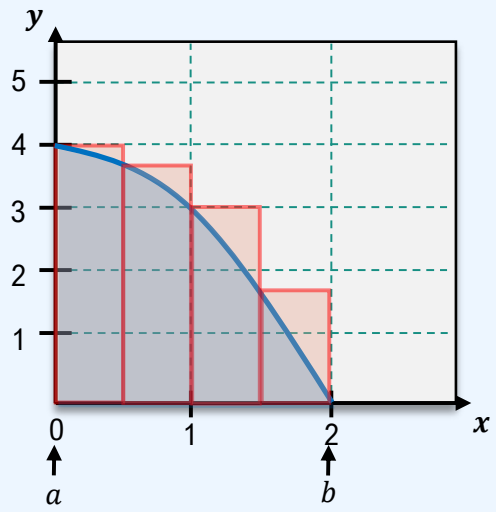
TOPIC: INTRODUCTION TO THE DEFINITE INTEGRAL

Definition of the Definite Integral

◆ Recall: The more subintervals (n) used, the more accurate the Riemann sum estimate for the area under a curve.

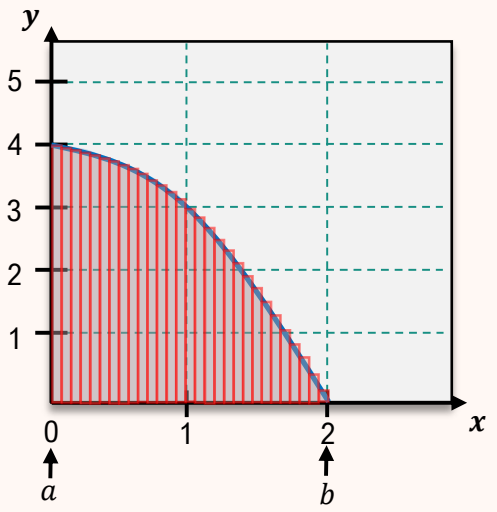
► Taking the limit as n approaches _____ gives the *true* area under the curve. This is called the **definite integral**.

Recall **Riemann Sum**



$$A \approx \sum_{k=1}^{n} f(x_k^*)\Delta x$$

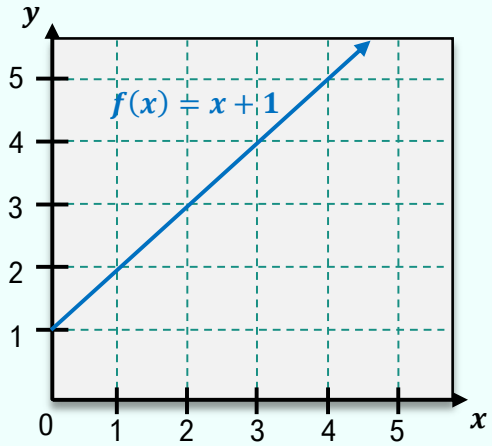
New **Definite Integral**



$$\lim_{\text{lower bound}}^{\text{upper bound}} \sum_{k=1}^{n} f(x_k^*)\Delta x = \int f(x)\,dx$$

EXAMPLE Express the following limit as a definite integral on the interval [0,4] and find the *exact* area that it represents.

$$\lim_{n\to\infty} \sum_{k=1}^{n} (x_k^* + 1)\Delta x$$



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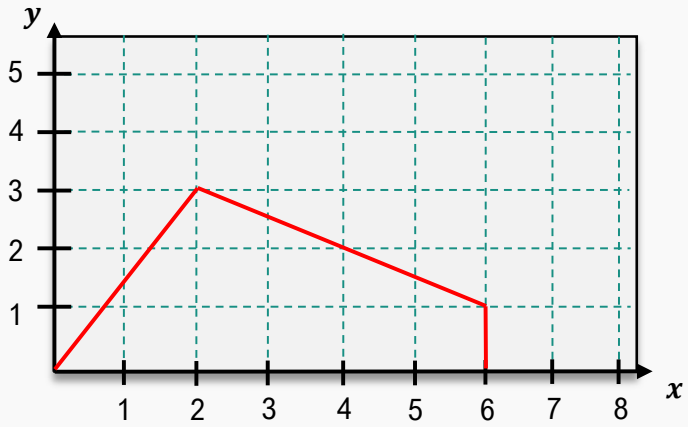
PRACTICE Find $\int_0^6 f(x)\,dx$ given the graph of $y = f(x)$.



PRACTICE Express the following limit as a definite integral on the interval $[0,10]$.

$$\lim_{n\to\infty}\sum_{k=1}^{n}(x_k^* - 3)^2\Delta x$$

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Basic Rules for Definite Integrals

◆ The basic rules that apply to indefinite integrals also apply to definite integrals.

► Definite integrals have *additional* rules related to their _____.

RULES OF INTEGRATION		
Name	**Rule**	**Example**
Sum & Difference	$\int_a^b [f(x) \pm g(x)]\,dx = \int_{\_}^{\_} f(x)\,dx \pm \int_{\_}^{\_} g(x)\,dx$	$\int_{-2}^{5} (x+6)\,dx = \int_{\_}^{\_} x\,dx + \int_{\_}^{\_} 6\,dx$
Constant Multiple	$\int_a^b kf(x)\,dx = k\int_{\_}^{\_} f(x)\,dx$	$\int_2^6 5x^3\,dx = 5\int_{\_}^{\_} x^3\,dx$
Order of Integration	$\int_b^a f(x)\,dx = \_\_\_\_\int_a^b f(x)\,dx$	$\int_4^1 x^3\,dx = \_\_\_\_\int_{\_}^{\_} \_\_\_\_dx$
Zero Width Interval	$\int_a^a f(x)\,dx =$ _____, if $f(a)$ exists	$\int_3^3 (3x^2 - 4x)\,dx = \_\_\_$
Additivity	$\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_{\_}^{\_} \_\_\_\_\,dx, \quad a < c < b$	$\int_1^3 5x^4\,dx + \int_3^7 5x^4\,dx = \int \quad dx$

◆ Use *multiple* rules together to find integrals of more complicated functions.

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PRACTICE

Given the following definite integral of the function $f(x) = 3x^2 - 2x$, write the simplified integral $-\int_4^0 f(x)dx$.

PRACTICE

Write the two definite integrals subtracted below as a single integral.

$$\int_1^6 \sqrt{x^2 - 5x}\, dx - \int_{10}^6 \sqrt{x^2 - 5x}\, dx$$

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PRACTICE Evaluate the following definite integral.

$$\int_{100}^{100} \frac{\sin(2x) - 100x^2}{\sqrt{x^5 + \tan(3x - 6)}}\, dx$$