TOPIC: TANGENT LINES AND DERIVATIVES

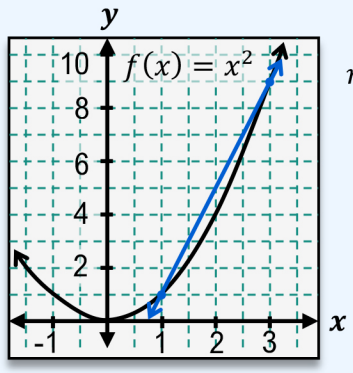
Slopes of Tangent Lines

- Recall: **Secant lines** *intersect* the curve at **[1 | 2]** point(s). Slope = **[AVERAGE | INSTANTANEOUS]** rate of change.
  - **Tangent lines** _____ the curve at **[1 | 2]** point(s). Slope = **[AVERAGE | INSTANTANEOUS]** rate of change.
    - The slope of a tangent line is also called the **derivative**.

EXAMPLE Given $f(x) = x^2$, find the slope of the tangent line at $x = 1$.

Recall — Secant (Average RoC)

$$m_{\text{sec}} = \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$



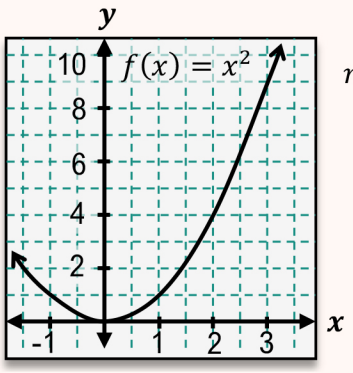
$$m_{\text{sec}} = \frac{f(3) - f(1)}{3 - 1}$$

$$= \frac{3^2 - 1^2}{3 - 1}$$

$$= \frac{9 - 1}{2} = \frac{8}{2} = 4$$

New — Tangent (Instantaneous RoC)

$$m_{\tan} = \frac{f(x) - f(c)}{x - c} \qquad c = \_\_\_$$



$$m_{\tan} = \frac{f(\quad) - f(1)}{-1}$$

$$= \lim_{x \to 1}$$

$$= \lim_{x \to 1}$$

$$= \lim_{x \to 1}$$

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PRACTICE Given the function $f(x) = 4x^2 - 1$, calculate the slope of the tangent line at $x = -3$.

PRACTICE Given the function $f(x) = x^2 - 10x + 2$, calculate the slope of the tangent line at $x = 2$.

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PRACTICE Given the function $f(x) = x^2 + 100$, calculate the slope of the tangent line at $x = 0$.

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Equations of Tangent Lines

Recall

$$m_{\tan} = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

- Recall: Tangent lines just *touch* the curve at 1 point.
  - To find eq'n of a tan line, find _____ then plug into point-slope form.

EXAMPLE Find the eq'n of the line tangent to $f(x) = 3x^2 - 4$ at $x = -2$.

x-val $(c) =$ ___ y-val $(f(c)) =$ ___

$m_{\tan} = \lim_{x \to}$

$= \lim_{x \to}$

$= \lim_{x \to}$

$= \lim_{x \to}$

$= \lim_{x \to}$

$= \lim_{x \to}$

HOW TO: Find the Tangent Line to a Curve at a Point

1) Plug x-val (__) into $f(x)$ to get y-val $(f(c))$ *(if needed)*

2) Plug given $f(x)$, c, and $f(c)$ into

$$m_{\tan} = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

3) Evaluate limit *(might have to _____ & cancel)*

4) Plug $m_{\tan}$ & point into

$$(y - y\text{-val}) = m_{\tan}\,(x - x\text{-val})$$

5) Solve equation for y

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PRACTICE Given the function $f(x) = 3(x^2 - 1)$, find the equation of the tangent line at $x = 1$.

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EXAMPLE Find the equation of the line tangent to $f(x) = -4x^2$ at $x = -2$.

$m_{\tan} = \lim_{x\to}$

$= \lim_{x\to}$

$= \lim_{x\to}$

$= \lim_{x\to}$

$= \lim_{x\to}$

$= \lim_{x\to}$

$= \lim_{x\to}$