## TOPIC: IMPROPER INTEGRALS

### Improper Integrals: Infinite Intervals

◆ An integral with _____ as one (or both) bounds is a type of *improper* integral.

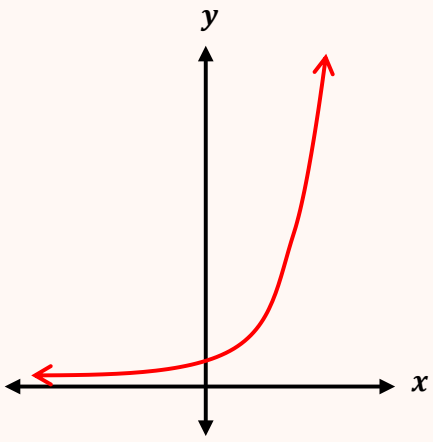
**New** **Improper Integral with Infinite Bounds**

If $f(x)$ is **continuous** on… then…

(1) $[a, \infty)$ $\int_a^{+\infty} f(x)\,dx =$ _____

(2) $(-\infty, b]$ $\int_{-\infty}^{b} f(x)\,dx =$ _____

(3) $(-\infty, \infty)$ $\int_{-\infty}^{+\infty} f(x)\,dx =$ _____ + _____




◆ An integral is *convergent* if the limit _____ (a finite number) and *divergent* if the limit _____.

**EXAMPLE** Determine whether each improper integral is convergent or divergent.

**New**

$$\int_a^{+\infty} f(x)\,dx = \lim_{t\to\infty} \int_a^t f(x)\,dx$$

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{t\to-\infty} \int_t^b f(x)\,dx$$

$$\int_{-\infty}^{+\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_c^{\infty} f(x)\,dx$$

**(A)** $\int_0^{\infty} e^x\,dx$

**[ CONVERGENT | DIVERGENT ]**

**(B)** $\int_{-\infty}^{0} e^x\,dx$

**[ CONVERGENT | DIVERGENT ]**

**(C)** $\int_{-\infty}^{\infty} e^x\,dx$

**[ CONVERGENT | DIVERGENT ]**

## TOPIC: IMPROPER INTEGRALS

**EXAMPLE** Evaluate the integral or state that it diverges.

$$\int_{-\infty}^{0} x \sin x \; dx$$

**EXAMPLE** Determine whether the improper integral diverge or converge. Evaluate the integral if it converges.

$$\int_{0}^{\infty} \frac{2}{1+x^2} \; dx$$

## TOPIC: IMPROPER INTEGRALS

**EXAMPLE** Determine whether the improper integrals diverge or converge. Evaluate the integral if it converges.

**(A)**

$$\int_0^{\infty} x\, e^x\, dx$$

**(B)**

$$\int_{-\infty}^{0} xe^x\, dx$$

**(C)**

$$\int_{-\infty}^{\infty} xe^x\, dx$$

## TOPIC: IMPROPER INTEGRALS

**PRACTICE** Evaluate the integral or state that it diverges.

($\boldsymbol{A}$)

$$\int_{-\infty}^{-1} \frac{2}{x^3}\, dx$$

---

($\boldsymbol{B}$)

$$\int_{2}^{\infty} \frac{1}{x(\ln x)^4}\, dx$$