TOPIC: INVERSE TRIG FUNCTIONS & BASIC TRIG EQUATIONS

Inverse Cosine

◆ Just like a square root *undoes* a square, the **inverse cosine** function (or arccos) *undoes* the cosine function.

Recall | **Inverse Cosine**

$\cos\frac{\pi}{6}$

Operation →

$\frac{\pi}{6}$ ***Angle*** ⇄ $\frac{\sqrt{3}}{2}$ ***Value***

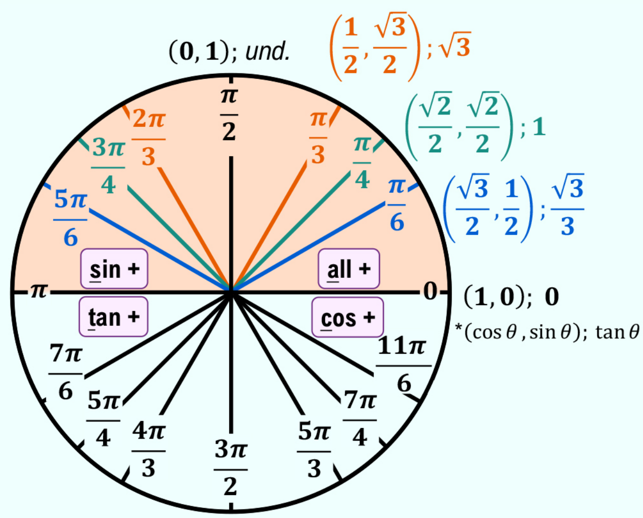
← *Inverse*

$\cos^{-1}\frac{\sqrt{3}}{2}$

◆ To evaluate an $\cos^{-1}$ function, find the ________ with the corresponding value on the unit circle.

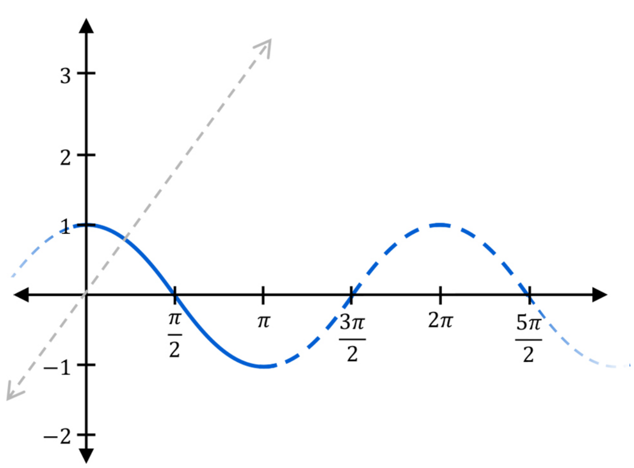
EXAMPLE Evaluate the expression.

$\cos^{-1}\frac{1}{2}$



◆ Since $\cos(x)$ is not __________, $\cos^{-1}(x)$ is only defined for *certain values* so that it passes the vertical line test.

- Recall: An *inverse* function is a reflection over the line _________.
- So, when taking $\cos^{-1}(x)$, only use angles in the interval [____ , ____].



New

Input: ____ $< x <$ ____

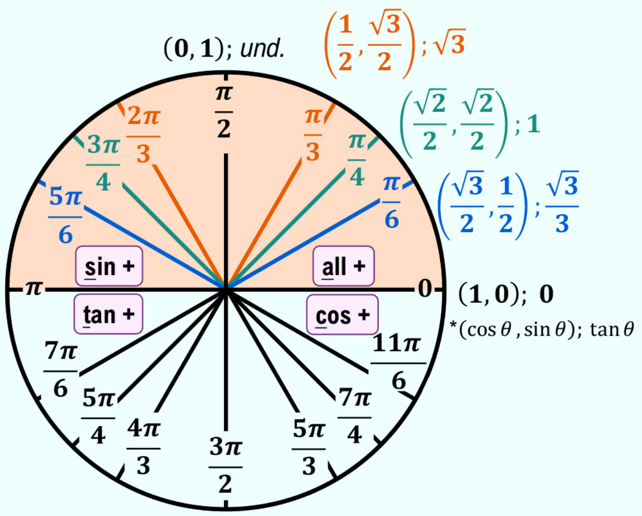
Output: ____ $< \theta <$ ____

(Inverse Cosine Interval)

TOPIC: INVERSE TRIG FUNCTIONS & BASIC TRIG EQUATIONS

EXAMPLE Evaluate the expression.

$$\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$



PRACTICE Evaluate the expression.

(**A**) $\cos^{-1}(-1)$

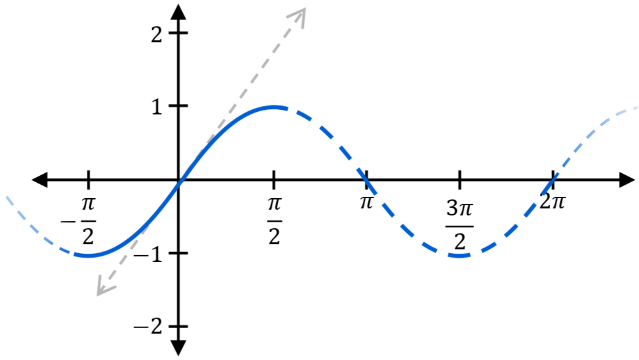
(**B**) $\cos^{-1} 0$

(**C**) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

TOPIC: INVERSE TRIG FUNCTIONS & BASIC TRIG EQUATIONS

Inverse Sine

◆ Like inverse cosine, $\sin^{-1}(x)$ is only defined for *certain values* because the sine function is not one-to-one.



New

Input: ____ $< x <$ ____

Output: ____ $< \theta <$ ____

(Inverse Sine Interval)

◆ To evaluate an inverse sine function, find the ∠ with the corresponding value on the unit circle within the interval.

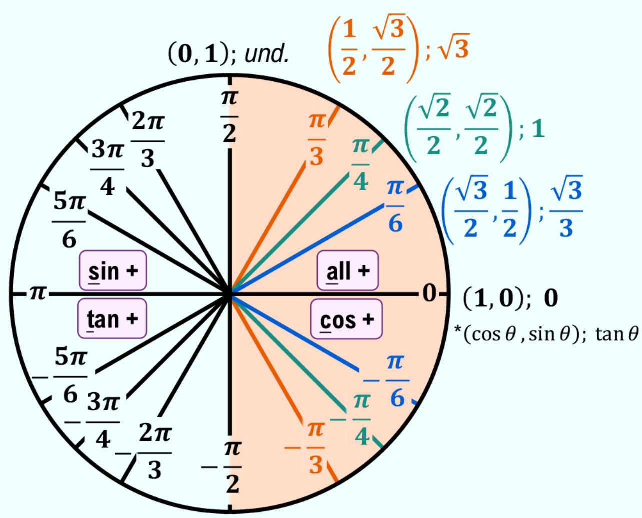
EXAMPLE Evaluate the expression.

(***A***)

$$\sin^{-1}\frac{1}{2}$$

(***B***)

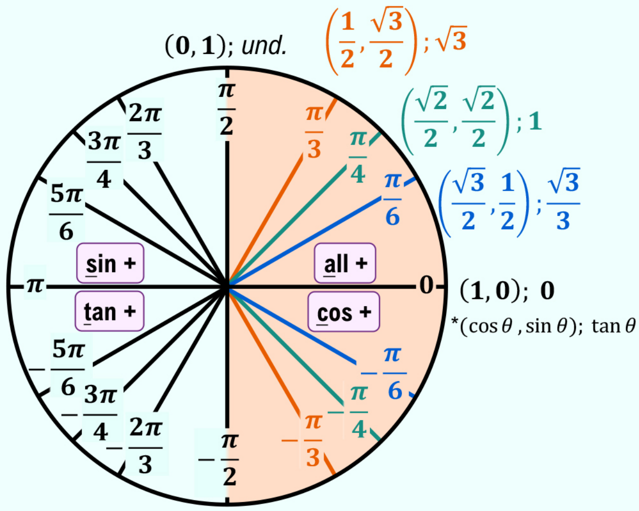
$$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$$



TOPIC: INVERSE TRIG FUNCTIONS & BASIC TRIG EQUATIONS

EXAMPLE Evaluate the expression.

$\sin^{-1}(-1)$



PRACTICE Evaluate the expression.

(A) $\sin^{-1} 1$

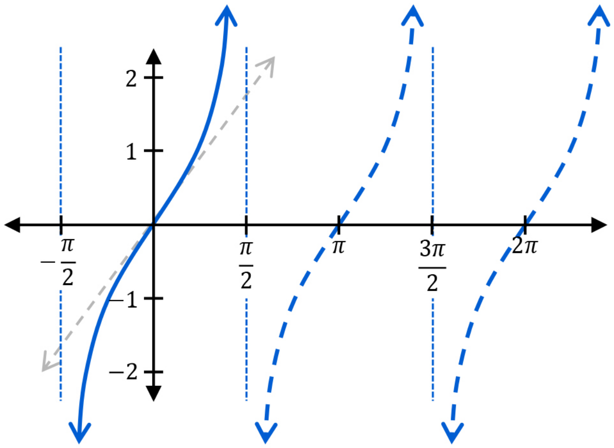
(B) $\sin^{-1} \frac{\sqrt{3}}{2}$

(C) $\sin^{-1} \left(\frac{\sqrt{2}}{2}\right)$

TOPIC: INVERSE TRIG FUNCTIONS & BASIC TRIG EQUATIONS

Inverse Tangent

◆ Like inverse sine & cosine, $\tan^{-1}(x)$ is only defined for *certain values* because the tangent fn is not one-to-one.



New

Input: ____ $< x <$ ____

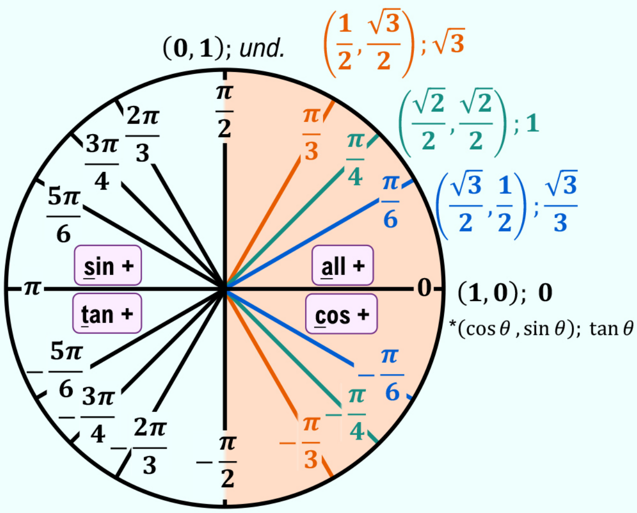
Output: ____ $< \theta <$ ____

(Inverse Tangent Interval)

EXAMPLE Evaluate the expression.

(***A***) $\tan^{-1}\sqrt{3}$

(***B***) $\tan^{-1}(-1)$

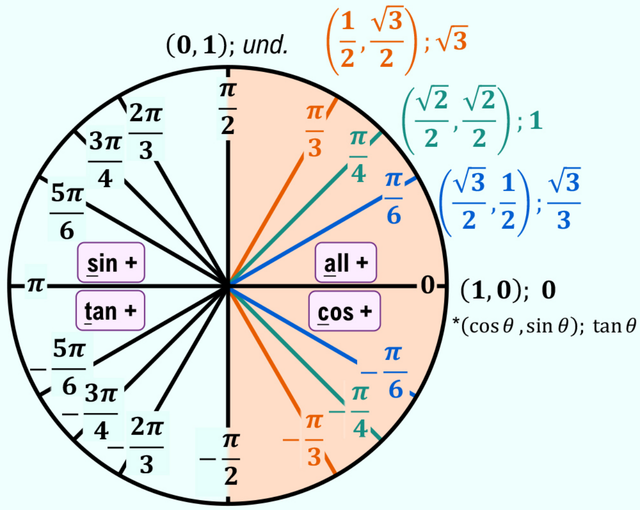


◆ To evaluate inverse trig functions using a calculator, press 2nd → SIN, COS or TAN. *(usually in RAD mode)*

TOPIC: INVERSE TRIG FUNCTIONS & BASIC TRIG EQUATIONS

EXAMPLE Evaluate the expression.

$\tan^{-1}(-\sqrt{3})$



PRACTICE Evaluate the expression.

(A) $\tan^{-1} 0$

(B) $\tan^{-1} 1$

(C) $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

PRACTICE Evaluate the expression using a calculator. Express your answer in radians, rounding to two decimal places.

(A) $\tan^{-1}(5)$

(B) $\sin^{-1}\left(-\frac{1}{3}\right)$

(C) $\cos^{-1}\left(\frac{1}{4}\right)$

TOPIC: EVALUATE COMPOSITE TRIG FUNCTIONS

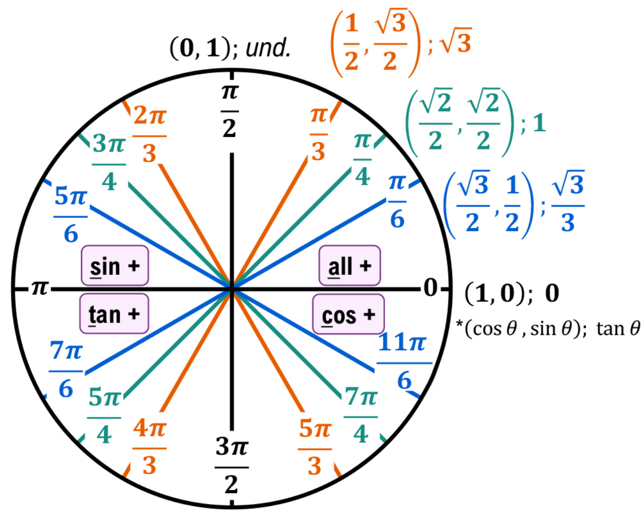
Evaluate Composite Functions - Values on Unit Circle

◆ To fully evaluate composite trig functions, evaluate function *inside* ________________ *first*.

$$\cos\theta = \frac{1}{2}$$

$$\sin\left(\cos^{-1}\frac{1}{2}\right)$$

$$\sin\left(\underline{\qquad\qquad}\right)$$



◆ When working with inverse trig functions, remember to *ALWAYS* use values in the correct interval.

EXAMPLE Evaluate the expression.

$(\boldsymbol{A})$

$\cos(\tan^{-1} 0)$

Recall Inverse Trig Intervals

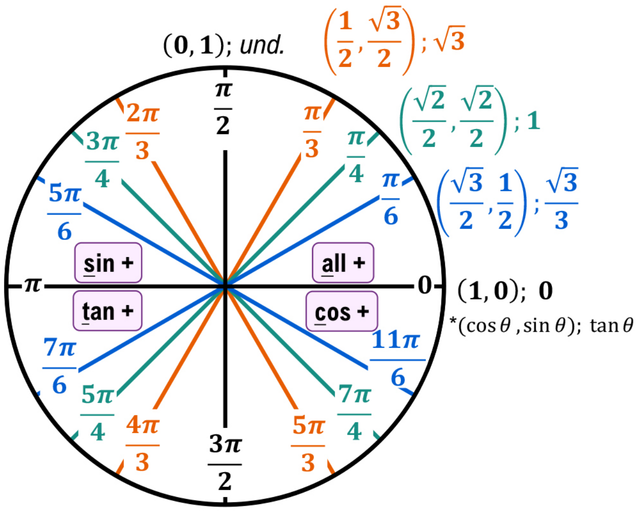
Function	Interval
$\cos^{-1}$	$[-1,1]$ / $[0,\pi]$
$\sin^{-1}$	$[-1,1]$ / $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$\tan^{-1}$	$[-\infty,\infty]$ / $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$(\boldsymbol{B})$ $\cos^{-1}\left(\sin\frac{\pi}{3}\right)$

TOPIC: EVALUATE COMPOSITE TRIG FUNCTIONS

Recall Inverse Trig Intervals

Function	Interval
$\cos^{-1}$	$[-1,1]$ / $[0,\pi]$
$\sin^{-1}$	$[-1,1]$ / $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$\tan^{-1}$	$[-\infty,\infty]$ / $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$



PRACTICE Evaluate the expression.

(A) $\cos(\sin^{-1} 1)$

(B) $\sin^{-1}\left(\cos\frac{2\pi}{3}\right)$

EXAMPLE Evaluate the expression.

$\sec\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

TOPIC: EVALUATE COMPOSITE TRIG FUNCTIONS

Evaluate Composite Functions – Special Cases

- Though trig functions & their inverse undo each other, you CANNOT assume the argument is your final answer.

EXAMPLE Evaluate the expression.

(A)

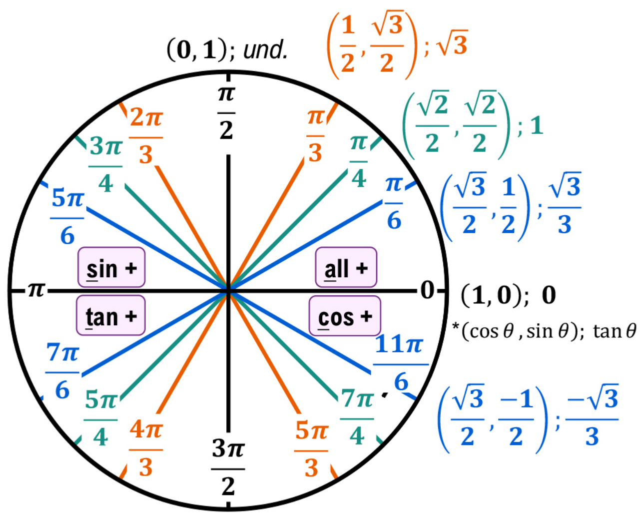
$$\cos^{-1}\left(\cos\frac{11\pi}{6}\right)$$

(B)

$$\sin(\sin^{-1}(2))$$

Recall Inverse Trig Intervals

Function	Interval
$\cos^{-1}$	$[-1,1]$ / $[0,\pi]$
$\sin^{-1}$	$[-1,1]$ / $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$\tan^{-1}$	$[-\infty,\infty]$ / $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$



EXAMPLE Evaluate the expression.

$$\sin^{-1}\left(\sin\frac{\pi}{6}\right)$$

PRACTICE Evaluate the expression.

(A) $\cos(\cos^{-1}(-\sqrt{3}))$

(B) $\cos^{-1}\left(\cos\left(\frac{\pi}{2}\right)\right)$

(C) $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$