TOPIC: SUBSTITUTION

Indefinite Integrals

Recall

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

◆ Recall: To evaluate *derivatives* of composite functions, we used the chain rule:

► To evaluate *integrals* with composite functions, use **substitution**.

New **Substitution**

$$\int f(g(x)) \cdot g'(x)\ dx = \int f(\_\_\_)\ \_\_\_\_$$

$$\int (x^2 + 1)^3 \cdot 2x\, dx = \int \_\_\_\_\_\_\_\_\_\_$$

$$= \_\_\_\_\_\_\_\_\_\_\_\_$$

$$= \_\_\_\_\_\_\_\_\_\_\_\_$$

EXAMPLE Evaluate the integral by making a substitution.

$$\int \sqrt{4x-1}\, dx = \int \sqrt{4x-1} \cdot \_\_\_\_\_\ dx$$

HOW TO: Evaluate Indefinite Integral with Substitution

1) Choose $u = g(x)$ (**inside** fcn), then find $du = g'(x)\,dx$

2) Rewrite int. ***only*** in terms of u & du; *If needed*: Mult. by ______ & ______

3) Integrate with respect to u

4) Replace u with $g(x)$

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EXAMPLE Use substitution to evaluate the given integral. Check your answer by differentiating.

$$\int (2x^3 + x + 7)^5 \, (6x^2 + 1) \, dx$$

HOW TO: Evaluate Indefinite Integral with Substitution

1) Choose $u = g(x)$ (**inside** fcn), then find $du = g'(x)\,dx$

2) Rewrite int. ***only*** in terms of u & du; *If needed*: Mult. by constant & reciprocal

3) Integrate with respect to u

4) Replace u with $g(x)$

EXAMPLE Evaluate the indefinite integral.

$$\int \cos t \cdot \sin^{99} t \; dt$$

TOPIC: SUBSTITUTION

PRACTICE Evaluate the indefinite integral.

HOW TO: Evaluate Indefinite Integral with Substitution

1) Choose $u = g(x)$ (**inside** fcn), then find $du = g'(x)\,dx$

2) Rewrite int. ***only*** in terms of u & du; *If needed*: Mult. by constant & reciprocal

3) Integrate with respect to u

4) Replace u with $g(x)$

$(\boldsymbol{A})$

$$\int 3t\sqrt{t^2+7}\,dt$$

$(\boldsymbol{B})$

$$\int \frac{1}{(3x+2)^5}\,dx$$

$(\boldsymbol{C})$

$$\int \theta \cdot \sec^2(5\theta^2+1)\ d\theta$$

$(\boldsymbol{D})$

$$\int \cos^5 x \sin^3 x\ dx$$

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Substitution with an Extra Variable

◆ Recall: If du is missing a constant multiple, multiply by that constant & its reciprocal to make substitution work.

► If the integrand has an "extra x", rearrange $u = g(x)$ to get ___ in terms of ___ & replace in integral.

EXAMPLE Evaluate the integral by making a substitution.

$$\int x\sqrt{x+3}\ dx$$

HOW TO: Evaluate Indefinite Integral with Substitution

1) Choose $u = g(x)$ (**inside** fcn), then find $du = g'(x)\ dx$

2) Rewrite int. ***only*** in terms of u & du;

If needed: ► Mult. by constant & recip.

► Rewrite x in terms of u

3) Integrate with respect to u

4) Replace u with $g(x)$

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PRACTICE Evaluate the indefinite integral.

HOW TO: Evaluate Indefinite Integral with Substitution

1) Choose $u = g(x)$ (**inside** fcn), then find $du = g'(x)\,dx$

2) Rewrite int. ***only*** in terms of u & du;
If needed: ► Mult. by constant & recip.
► Rewrite x in terms of u

3) Integrate with respect to u

4) Replace u with $g(x)$

$(\boldsymbol{A})$

$$\int x(5+x)^{79}dx$$

$(\boldsymbol{B})$

$$\int \frac{t}{\sqrt{t-2}}dt$$

$(\boldsymbol{C})$

$$\int \frac{x}{(x-6)^5}dx$$

$(\boldsymbol{D})$

$$\int \sin\theta\,(1+\sin\theta)^{84}\cos\theta\,d\theta$$

Hint: $sin^2\,x = 1 - cos^2\,x$

TOPIC: SUBSTITUTION

Definite Integrals

◆ To find *definite* integrals using substitution, there are two methods you can use:

► Method 1: Use substitution to solve as indefinite integral, then evaluate at *original* bounds.

EXAMPLE Evaluate the integral by making a substitution.

(A)

$$\int_0^2 (x^2+1)^3 \cdot 2x\,dx$$

$$\int \underbrace{(x^2+1)}_{u}{}^3 \cdot \underbrace{2x\,dx}_{du} = \int u^3\,du$$

HOW TO: Evaluate Definite Integrals with Substitution – Method 1

1) Choose $u = g(x)$ (**inside** fcn), then find $du = g'(x)\,dx$

2) Rewrite int. ***only*** in terms of u & du;
If needed: ► Mult. by constant & recip.
► Rewrite x in terms of u

3) Integrate with respect to u

4) Replace u with $g(x)$

5) Evaluate antiderivative at **original** bounds

► Method 2: Rewrite integrand in terms of u & du, solve definite integral, evaluating at *new* bounds $g(a)$ & $g(b)$.

New

$$\int_{\_\_}^{\_\_} f(g(x)) \cdot g'(x)\;dx = \int_{\_\_}^{\_\_} f(u)\,du$$

(B)

$$\int_0^2 \underbrace{(x^2+1)}_{u}{}^3 \cdot \underbrace{2x\,dx}_{du} = \int_{\_\_}^{\_\_} u^3\,du$$

HOW TO: Evaluate Definite Integrals with Substitution – Method 2

1) Choose $u = g(x)$ (**inside** fcn), then find $du = g'(x)\,dx$

2) a. Rewrite int. ***only*** in terms of u & du;
If needed: ► Mult. by constant & recip.
► Rewrite x in terms of u

b. Transform bounds: plug into $u = g(x)$

3) Integrate with respect to u

~~4) Replace u with $g(x)$~~

4) Evaluate antiderivative at **new** bounds

TOPIC: SUBSTITUTION

PRACTICE Evaluate the definite integral.

(**A**)

$$\int_0^1 \frac{t}{\sqrt{t^2+1}}\, dt$$

(**B**)

$$\int_1^2 (x-3)(x^2-6x)^7\, dx$$

(**C**)

$$\int_{-\pi/4}^{\pi/4} \tan y \sec^2 y\ dy$$