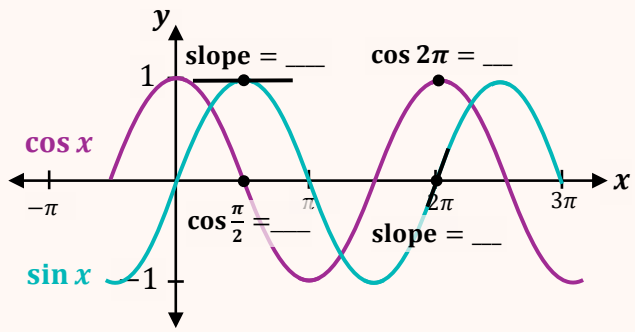
TOPIC: DERIVATIVES OF TRIG FUNCTIONS

Derivatives of Sine & Cosine

◆ Instead of using limits, use these rules to quickly find derivatives of functions with **sin** & **cos**.

New **Derivatives of Sine & Cosine**



$$\frac{d}{dx}\sin x = \_\_\_\_\_$$

$$\frac{d}{dx}\cos x = \_\_\_\_\_$$

EXAMPLE Find the derivative.

(A)

$$f(x) = 3x + \cos x$$

(B)

$$f(x) = x^2 \sin x$$

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PRACTICE Find the derivative of the function.

(***A***)

$$h(t) = \sin t \ \cos t$$

(***B***)

$$f(x) = \frac{5\cos x}{2x^3}$$

(***C***)

$$y = \frac{\sin\theta}{2 + \cos\theta}$$

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Higher Order Derivatives of Sine & Cosine

◆ To find $\frac{d^n}{dx^n}\sin x$ or $\frac{d^n}{dx^n}\cos x$, divide n by _____, then take derivatives based on _____.

EXAMPLE Find the indicated derivative.

Recall

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

$(\boldsymbol{A})$

$$\frac{d^{37}}{dx^{37}}\sin x$$

$$\frac{d}{dx}\ldots =$$

$$\frac{d^2}{dx^2}\ldots =$$

$$\frac{d^3}{dx^3}\ldots =$$

$$\frac{d^4}{dx^4}\ldots =$$

$(\boldsymbol{B})$

$$\frac{d^{58}}{dx^{58}}\cos x$$

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PRACTICE Find the indicated derivative.

(***A***) $f^{(19)}(x)$ of $f(x) = \cos x$

(***B***) $\dfrac{d^{1002}}{dx^{1002}}(\sin x)$

(***C***) $\dfrac{d^{52}y}{dx^{52}}$ of $y = \cos x$

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Derivatives of Other Trig Functions

◆ Derivatives of other trig functions can be found by rewriting in terms of $\sin$ & $\cos$, then using the quotient rule.

New **Derivative of Tangent**

$$\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x\,(\qquad) - \sin x\,(\qquad)}{\cos^2 x}$$

$$= \frac{\qquad\qquad\qquad\qquad}{\cos^2 x}$$

$$= \frac{\qquad}{\cos^2 x}$$

$$\frac{d}{dx}(\tan x) = \_\_\_\_\_$$

EXAMPLE Find the derivative.

Recall

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

New

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = \quad \csc^2 x$$

$$\frac{d}{dx}\sec x = \tan x \sec x \qquad \frac{d}{dx}\csc x = \quad \cot x \csc x$$

(*A*) $f(x) = 3x^2 + \cot x$

(*B*) $f(x) = 4x \sec x$

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PRACTICE Find the derivative of the function.

(***A***)

$$r = \csc x - 3\sin x + \tan x$$

(***B***)

$$f(x) = 4x^2 \sec x - \sqrt{x}$$

(***C***)

$$y = \frac{\cot\theta}{3 + \sec\theta}$$

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EXAMPLE Find the slope of the tangent line of $f(x)$ at $x = \frac{\pi}{4}$.

$f(x) = \sec x$