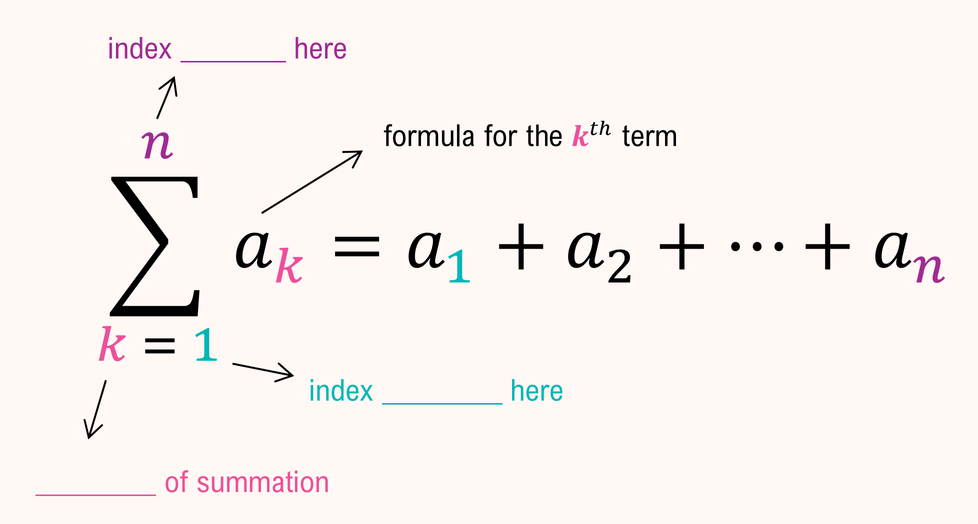
TOPIC: RIEMANN SUMS

Sigma Notation

◆ We can write a sum involving many terms in a more ____________ way using **sigma notation**.

New | **Sigma Notation**



◆ To evaluate a sum, plug in integer values, from the start index to the end index, into a_k, adding all terms together.

► The index of summation (k) can be represented by *any* letter and can start at *any* integer.

EXAMPLE Evaluate the following finite sums.

$(\boldsymbol{A})$

$$\sum_{k=1}^{3} k^2$$

$(\boldsymbol{B})$

$$\sum_{i=0}^{4} i + 3$$

TOPIC: RIEMANN SUMS

PRACTICE Evaluate the following summation:

$(\boldsymbol{A})$

$$\sum_{i=1}^{4} i^2 - 3i + 8$$

$(\boldsymbol{B})$

$$\sum_{k=1}^{4} \left(\frac{k}{2}\right)^2$$

$(\boldsymbol{C})$

$$\sum_{i=0}^{2} \frac{2i}{3}$$

TOPIC: RIEMANN SUMS

PRACTICE Evaluate the following summation (make sure your calculator is in radian mode):

$$\sum_{i=1}^{4} \sin\left(\frac{\pi i}{8}\right)$$

TOPIC: RIEMANN SUMS

EXAMPLE Write the following expanded sums in sigma notation:

$(\boldsymbol{A})$ $3 \bullet 1^2 + 3 \bullet 2^2 + 3 \bullet 3^2 + 3 \bullet 4^2 + 3 \bullet 5^2$

$(\boldsymbol{B})$ $\frac{3}{4} + \frac{3}{2} + \frac{9}{4} + 3$

TOPIC: RIEMANN SUMS

Algebra Rules for Finite Sums

◆ Just like for derivatives and indefinite integrals, there are sum/difference and constant rules for sums.

RULES FOR FINITE SUMS		
Name	**Rule**	**Example**
Sum & Difference	$\sum_{k=1}^{n}(a_k \pm b_k) = \sum_{k=1}^{n} a_k \quad \sum_{k=1}^{n} b_k$	$\sum_{k=1}^{3}(k^2 - k) = \sum_{k=1}^{3}\_\_ \quad \sum_{k=1}^{3}\_\_\_ =$
Constant Multiple	$\sum_{k=1}^{n} c \cdot a_k = \_\_\_ \cdot \sum_{k=1}^{n} a_k$	$\sum_{k=1}^{3} 2k^2 = \_\_ \cdot \sum_{k=1}^{3}\_\_\_ =$
Constant Value	$\sum_{k=1}^{n} c = c \cdot \_\_\_\_$	$\sum_{k=1}^{18} 3 = \_\_\_ \cdot \_\_\_\_ = \_\_\_\_$

◆ Use *multiple* rules together to find summations of more complicated functions.

EXAMPLE Evaluate the given sum.

$$\sum_{k=1}^{3}(6k^2 - 9)$$

TOPIC: RIEMANN SUMS

PRACTICE Evaluate the following summations:

$(\boldsymbol{A})$

$$\sum_{t=2}^{6} 5t + \frac{4}{t}$$

$(\boldsymbol{B})$

$$\sum_{i=0}^{4} 2i + 1$$

$(\boldsymbol{C})$

$$\sum_{i=1}^{2} 4i^3 - 3i^2 + 2i - 1$$

TOPIC: RIEMANN SUMS

PRACTICE Evaluate the following summation (make sure your solution is in radians):

$$\sum_{k=0}^{5} 2\tan\left(\frac{\pi k}{3}\right) - \pi k$$

TOPIC: RIEMANN SUMS

Introduction to Riemann Sums

◆ Recall: We can estimate the area under the curve by breaking it into n rectangles (subintervals) of width $\Delta x = \frac{b-a}{n}$.

► We can use _____________ notation to represent the estimated area as a **Riemann sum**.

EXAMPLE Estimate the area under the curve using a left endpoint Riemann sum with 4 subintervals.

Recall **Estimating Area**

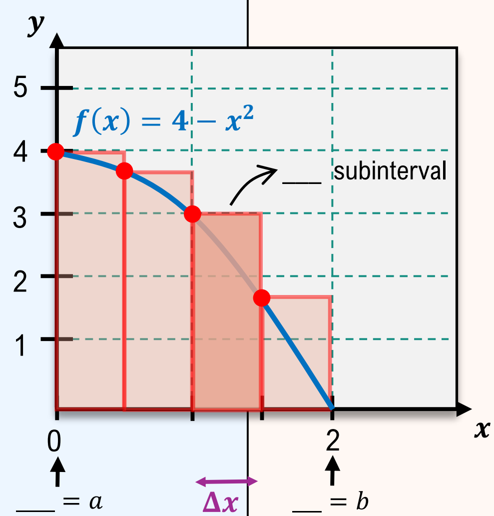
$A \approx A_1 + A_2 + A_3 + A_4$

$\approx w_1h_1 + w_2h_2 + w_3h_3 + w_4h_4$

$\approx \Delta x\big(f(0) + f(0.5) + f(1) + f(1.5)\big)$

$\approx (0.5)(4 + 3.75 + 3 + 1.75)$

≈ 6.25



New **Estimating Area with Riemann Sums**

$A \approx \sum_{k=1}^{n} f(\_\_\_\_)\Delta x$

(Left Riemann sum)

$A \approx \sum_{k=1} \qquad \Delta x$

$A \approx \_\_\_ \sum_{k=1}$

$\approx (\Delta x)$

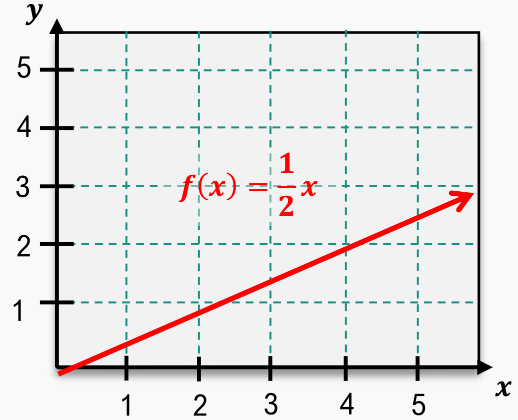
≈ 6.25

$A \approx \sum_{k=1}^{n} f(\_\_\_)\Delta x$

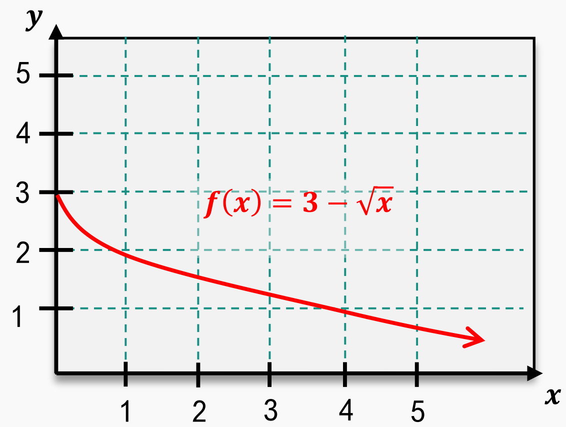
(General Riemann sum)

TOPIC: RIEMANN SUMS

PRACTICE For the following graph, write a Riemann sum using left endpoints to approximate the area under the curve over $[0,5]$ with 5 subintervals.

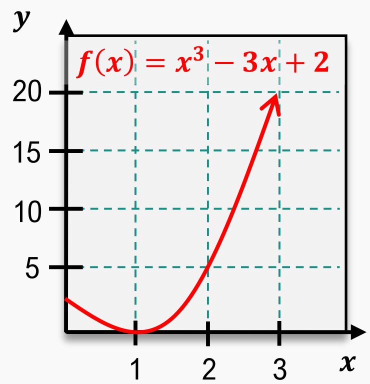


PRACTICE For the following graph, write a Riemann sum using left endpoints to approximate the area under the curve over $[0,6]$ with 6 subintervals.



TOPIC: RIEMANN SUMS

PRACTICE Write the Riemann sum that would approximate the area of the following graph over the interval $[0,3]$ using 3 subintervals.



TOPIC: RIEMANN SUMS

Left, Right, & Midpoint Riemann Sums

◆ Recall: When esimating area under a curve, we can use fcn values at the left endpoint, right endpoint, or midpoint.

► For Riemann sums, we can do the same thing.

EXAMPLE Set up the left, right, and midpoint Riemann sums for $f(x) = 4 - x^2$ on $[0,2]$.

Left, Right, & Midpoint Riemann Sums

Left

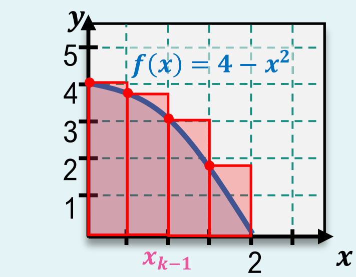
$x_k^* = x_{k-1}$ is the **left** endpoint of subinterval

$x_{k-1} = a + \underline{\hspace{2.5cm}}\Delta x$

Recall

$$A \approx L_n = \sum_{k=1}^{n} f(x_{k-1})\Delta x$$

(Left Riemann Sum)



$$L_n = \sum_{k=1}^{\underline{\quad}} (\underline{\hspace{3cm}})(\underline{\hspace{1cm}})$$

Right

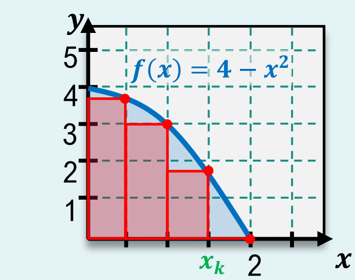
$x_k^* = \underline{\hspace{1.5cm}}$ is the **right** endpoint of subinterval

$x_k = a + \underline{\hspace{1.2cm}}\Delta x$

New

$$A \approx R_n = \sum_{k=1}^{n} f(\underline{\hspace{1.2cm}})\Delta x$$

(Right Riemann sum)



$$R_n = \sum_{k=1}^{\underline{\quad}} (\underline{\hspace{3cm}})(\underline{\hspace{1cm}})$$

Midpoint

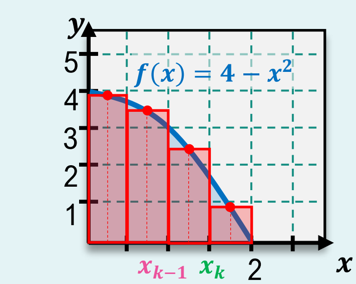
$x_k^* = \underline{\hspace{3.5cm}}$ is the **midpoint** of subinterval

$x_k^* = a + \underline{\hspace{2.5cm}}\Delta x$

New

$$A \approx M_n = \sum_{k=1}^{n} f\left(\underline{\hspace{3cm}}\right)\Delta x$$

(Midpoint Riemann sum)



$$M_n = \sum_{k=1}^{\underline{\quad}} (\hspace{4cm})(\underline{\hspace{1cm}})$$

TOPIC: RIEMANN SUMS

PRACTICE

Approximate the area under the curve $f(x) = \sqrt{x+3}$ over the interval $[1,5]$ using the Right Riemann sum with 8 subintervals.

PRACTICE

Approximate the area under the curve $f(x) = \frac{2}{3x+5}$ over the interval $[4,10]$ using the Midpoint Riemann sum with $n = 3$ subintervals.

TOPIC: RIEMANN SUMS

EXAMPLE

Set up and solve the left, right, and midpoint Riemann sums for $f(x) = x^3$ on the interval $[0,4]$. Use 4 subintervals. Tell which is an overestimate, and which is an underestimate.