MASTER TABLE: RULES OF DIFFERENTIATION

NOTE: This table spans multiple videos.

RULES OF DIFFERENTIATION		
Name	**Rule**	**Example**
General Exponential	$\frac{d}{dx}b^x = b^x \cdot \ln b$ $b > 0$, $b \neq 1$	$\frac{d}{dx}(6^x) =$
Natural Exponential	$\frac{d}{dx}e^x =$ ___ $\cdot \ln$ ___ $=$ _______	$\frac{d}{dx}(4e^x) =$
General Logarithmic	$\frac{d}{dx}\log_b x =$ ________ $b > 0$, $b \neq 1$, $x > 0$	$\frac{d}{dx}\log_8 x =$
Natural Logarithmic	$\frac{d}{dx}\ln x = \frac{d}{dx}\log_e x = \frac{1}{x \cdot \ln \text{\_\_\_\_\_}} =$ $x > 0$	$\frac{d}{dx}6\ln x =$

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

Derivatives of General Exponential Functions

◆ We can use limits to find a derivative rule that works for *all* exponential functions b^x.

New | **Derivative of b^x**

$$\frac{d}{dx}(b^x) = \lim_{h\to 0}\frac{b^{x+h} - b^x}{h}$$

$$= \lim_{h\to 0} \frac{\qquad}{}$$

$$= \quad \cdot \lim_{h\to 0} \frac{\qquad}{}$$

$$\frac{d}{dx}(b^x) = \_\_\_ \cdot \_\_\_\_\_$$

b ____
b ____

Recall

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

Limit Definition of a Derivative

$$\ln b = \lim_{h\to 0}\frac{b^h - 1}{h}$$

Limit Definition of ln at h=0

EXAMPLE Find the derivative of the following functions.

(A) $f(x) = 6^x$

(B) $g(x) = 3^{x^2+4x}$

◆ When taking the derivative of $f(x) = b^{g(x)}$, we can apply the **chain rule** to get $f'(x) = b^{g(x)} \cdot \ln b \cdot g'(x)$.

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

PRACTICE Find the derivative of the given function.

(***A***)

$f(x) = 2^x - 5^x$

(***B***)

$g(x) = x^4 + 4x + 4^x$

PRACTICE Find the derivative of the given function.

(***A***)

$y = (4x - 3x^2 + 9) \cdot 2^{5x}$

(***B***)

$h(x) = 4^{(\sqrt{x}+3x)^{\frac{5}{4}}}$

EXAMPLE Find the derivative of $f(\theta) = \sin\left(\theta \cdot 3^{\theta^2+4\theta}\right)$.

EXAMPLE

A medical lab uses a radioactive isotope for one of its tests. The quantity of the radioactive isotope, R, remaining in a sample after t hours is given by the function below.

$$R = 150\left(\frac{1}{3}\right)^{\frac{t}{12}}$$

$(\boldsymbol{A})$ Find the instantaneous rate of change $\frac{dR}{dt}$.

$(\boldsymbol{B})$ Compute the instantaneous rate of change after 12 hours, 1 day, and 2 days.

$(\boldsymbol{C})$ Interpret your results.

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

Derivatives of the Natural Exponential Function (e^x)

- Recall: $f(x) = e^x$ is just a special case of $f(x) = b^x$ where $b = e$.
  - We can use the derivative rule for general exponential functions to find the derivative of e^x.

RULES OF DIFFERENTIATION		
Name	**Rule**	**Example**
General Exponential	$\frac{d}{dx}b^x = b^x \cdot \ln b$ $b > 0$, $b \neq 1$	$\frac{d}{dx}(6^x) = 6^x \cdot \ln 6$
Natural Exponential	$\frac{d}{dx}e^x =$ ___ $\cdot \ln$ ___ $=$ ______	$\frac{d}{dx}(4e^x) =$

EXAMPLE Find the derivative of the following functions.

(A) $f(x) = 3e^{2x+4}$

(B) $g(x) = xe^{5x}$

- When taking the derivative of $f(x) = e^{g(x)}$, we can apply the **chain rule** to get $f'(x) = e^{g(x)} \cdot g'(x)$.

PRACTICE Find the derivative of the given function.

(A)

$f(x) = -3e^x + 5x - 2$

(B)

$g(x) = 7e^x + 2x^3$

EXAMPLE Find the derivative of the given function.

$y = e^{\sqrt{x^3 - 2x}}$

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

PRACTICE Find the derivative of the given function.

$(\boldsymbol{A})$

$$y = x^2 e^{3x^2+5x}$$

$(\boldsymbol{B})$

$$f(t) = \frac{e^{3t}}{t - 2e^{-t}}$$

$(\boldsymbol{C})$

$$y = \tan\left(e^{-x^3}\right)$$

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

EXAMPLE Find an equation of the tangent line to the function $y = 3e^x - x$ at $x = 0$.

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

Derivatives of General Logarithmic Functions

◆ Since $y = \log_b x$ is equivalent to __________, we can differentiate __________ sides of that equation to find $\frac{d}{dx}\log_b x$.

New **Derivative of $\log_b x$**

$y = \log_b x \quad \Leftrightarrow \quad x = b^y$

Recall $\frac{d}{dx} b^x = b^x \cdot \ln b$

$x = \quad b^y$

$1 = \_\_\_\_\_\_\_\_\_\_ \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\_\_\_\_\_\_\_\_\_\_} = \frac{1}{\_\_\_\_\_\_\_\_\_\_}$

$\frac{d}{dx}\log_b x = \_\_\_\_\_\_\_\_\_\_$ $\quad b > 0$, $b \neq 1$, $x > \_\_$

EXAMPLE Find the derivative of the following functions.

(A) $g(x) = \log_8 x$

(B) $f(x) = \log_5(x^2)$

◆ When taking the derivative of $f(x) = \log_b(g(x))$, we can apply the **chain rule** to get $f'(x) = \frac{1}{g(x) \cdot \ln b} \cdot g'(x)$.

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

PRACTICE Find the derivative of the given function.

(A)

$$f(x) = 8\log_2 x$$

(B)

$$g(t) = 2t + \log_5 t$$

EXAMPLE Find the derivative of $y = \log_4 \sqrt{\frac{x}{4x+1}}$.

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

PRACTICE Find the derivative of the given function.

$(\boldsymbol{A})$

$$f(x) = (x^3 + 2x) \cdot \log_5 x$$

$(\boldsymbol{B})$

$$g(t) = \log_5(7^{t^2+4})$$

$(\boldsymbol{C})$

$$h(x) = \sin\left(\log_4(x^2)\right)$$

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

Derivatives of the Natural Logarithmic Function

◆ Recall: $f(x) = \ln x$ is just a special case of $f(x) = \log_b x$ where $b = e$.

► We can use the derivative rule for general log functions to find the derivative of $f(x) = \ln x = \log\ \ x$.

RULES OF DIFFERENTIATION			
Name	**Rule**		**Example**
General Logarithmic	$\frac{d}{dx}\log_b x = \frac{1}{x \cdot \ln b}$	$b > 0$ $b \neq 1$ $x > 0$	$\frac{d}{dx}\log_8 x = \frac{1}{8 \ln 8}$
Natural Logarithmic	$\frac{d}{dx}\ln x = \frac{d}{dx}\log_e x = \frac{1}{x \cdot \ln \_\_\_\_}=$	$x > 0$	$\frac{d}{dx} 6 \ln x =$

EXAMPLE Find the derivative of the given function.

(A) $f(x) = \ln(x^2 + 4x)$

(B) $g(x) = x \ln x^3$

◆ When taking the derivative of $f(x) = \ln(g(x))$, we can apply the **chain rule** to get $f'(x) = \frac{1}{g(x)} \cdot g'(x)$.

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

PRACTICE Find the derivative of the given function.

$(\boldsymbol{A})$

$$f(x) = 2x^3 - 1 + \ln x$$

$(\boldsymbol{B})$

$$g(x) = e^x + \ln x^5$$

PRACTICE Find the derivative of the given function.

$(\boldsymbol{A})$

$$h(x) = \ln\left(\frac{\sqrt{x+1}}{x^2+3}\right)$$

$(\boldsymbol{B})$

$$y = x^2 \ln(x^2)$$

$(\boldsymbol{C})$

$$g(x) = e^{x^2 \ln x}$$

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

EXAMPLE Use implicit differentiation to find $\frac{dy}{dx}$, where $\ln(x^2 y) = 2e^{3x+y}$.

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

EXAMPLE

The time t, in days that it takes the number of bacteria in a sample to reach B is given by the function below. Find $t'(1000)$ and explain what it represents.

$$t(B) = 42\ln\left(\frac{650}{1500 - B}\right)$$

TOPIC: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

Graphical Applications of Exponential & Logarithmic Functions

EXAMPLE Find the critical points of the given function.

$y = \ln(x - 1)$

EXAMPLE Find the absolute maximum and minimum values of the function on the given interval.

$f(x) = x \ln x \, ; [1, 2]$

HOW TO: Find Global Extrema on a Closed Interval

1) Find critical points:

$f'(x) = 0$ or $f'(x)$ DNE

2) Plug critical pts (if in **interval**) & endpts into $\boldsymbol{f(x)}$

3) Of values found in **(2)**:

Largest = global **MAX**

Smallest = global **MIN**

Graphical Applications of Exponential & Logarithmic Functions

PRACTICE Identify the open intervals on which the function is increasing or decreasing.

$f(x) = xe^{-2x}$

HOW TO: Determine Intervals of Increase & Decrease

1) Find critical points:

$f'(x) = 0$ or $f'(x)$ DNE

2) Make sign chart **intervals** based on critical points

3) Plug value from each int. into f':

If $+$, f **INC** on interval

If $-$, f **DEC** on interval

PRACTICE Identify the local minimum and maximum values of the given function, if any.

$f(t) = t^2 \ln t \, , t > 0$

HOW TO: Find Local Extrema Using First Derivative Test

1) Find critical points:

$f'(x) = 0$ or $f'(x)$ DNE

2) Make sign chart intervals based on critical points

3) Plug value from each int. into f'

If f' changes from:

$+ \rightarrow -$, crit. pt. is local **MAX**

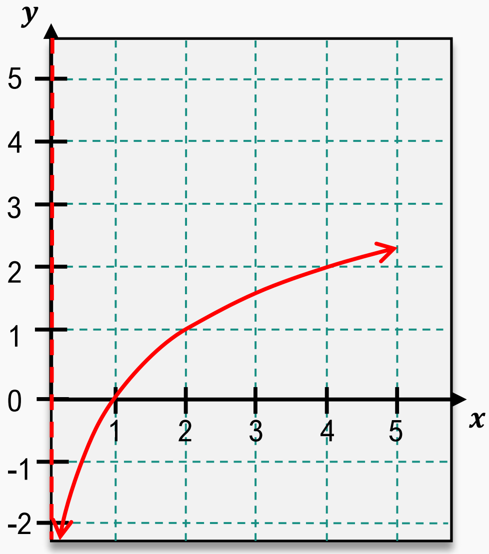
$- \rightarrow +$, crit. pt. is local **MIN**

4) If asked: Find **value** of max/min by plugging crit. pt. into $f(x)$

Graphical Applications of Exponential & Logarithmic Functions

PRACTICE

For the following graph, find the open intervals for which the function is concave up or concave down. Identify any inflection points.



PRACTICE

Determine the intervals for which the function is concave up or concave down. State the inflection points.

$f(x) = 4\ln(3x^2)$

HOW TO: Determine Intervals of Concavity

1) Find potential inflection points:

$f''(x) = 0$ or DNE

2) Make sign chart **intervals** based on potential **inflection points**

3) Plug value from each int. into f'':

If $+$, f concave **UP** on int.

If $-$, f concave **DOWN** on int.