TOPIC: SEPARABLE DIFFERENTIAL EQUATIONS

Separation of Variables

◆ A first-order DE is separable when $\frac{dy}{dx}$ *can* be written as the _____ of a function of x times a function of y.

► To separate variables, collect the function of y on one side w/ ___ & the function of x on the other side w/ ___.

EXAMPLE Separate the variables of the following differential equations.

(A)

$$\frac{dy}{dx} = (x^2 - 3)(6y^3)$$

(B)

$$\sqrt{x} - \sqrt{y}\frac{dy}{dx} = 0$$

(C)

$$2xy' - y\ln x^3 = 0$$

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EXAMPLE Separate the variables of the following differential equations, if possible.

(A)

$$\frac{dr}{dt} = e^{r-2t}$$

(B)

$$y' - (3 + x)(y^2 - 4) = 0$$

(C)

$$\frac{dy}{dx} = x + e^y + y^2$$

(D)

$$t^2 dG - \sqrt{G - 70}\, dt = 0$$

TOPIC: SEPARABLE DIFFERENTIAL EQUATIONS

PRACTICE Separate the variables of the following differential equation.

$$y \sin^2 \theta \frac{dy}{d\theta} = 1$$

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Solving Separable Differential Equations

◆ We can solve separable DEs by first _____ and then _____ both sides.

EXAMPLE

Find the particular solution of $y' = y(x + 2)$ that satisfies the initial condition $y(-2) = 1$.

$y' = y(x + 2)$

HOW TO: Solve Separable DEs

1) Separate the variables

2) Integrate **both** sides

**If solving for general sol'n, skip to 4)*

3) Find _____ by plugging in initial condition

4) Solve for y (not possible if *implicit*)

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EXAMPLE Find the general solution to the differential equation $(2 + x^2)y' - xy = 0$.

EXAMPLE Find the particular solution that satisfies the given initial condition $y' = e^{x-y}; y(0) = 2$.

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PRACTICE Find the particular solution that satisfies the given initial condition $(y^2 + y)e^x y' = y^3 + e^x y^3$; $y(0) = 1$.

PRACTICE Find the general solution to the differential equation $\frac{dy}{dx} = y\sqrt{x}$.

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PRACTICE Find the particular solution that satisfies the given initial condition $\frac{dy}{dx} = \sin x \cdot \sec y \, , y\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$.

TOPIC: SEPARABLE DIFFERENTIAL EQUATIONS

Exponential Growth and Decay

◆ Exponential change occurs when the _____ of y w/ respect to time t is proportional to the _____ of y.

New **Exponential Growth & Decay Model**

$$\frac{dy}{dt} = ky$$

Separate variables:

Integrate both sides:

Solve for y to get general solution:

$y =$ _____

_____ value ($t =$__) _____ constant

◆ Exponential *growth* occurs if k is _____ and exponential *decay* if k is _____.

EXAMPLE

Suppose a population of mice increases according to the law of exponential growth. The initial population of mice is 100 mice and after 6 days passed, the population had grown to 300. Approximately how many mice would there have been after only 3 days?

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EXAMPLE

A population of fruit flies increases exponentially in an experiment. There were 300 flies on the fourth day of the experiment, and 750 flies on the eighth day of the experiment. Approximately how many flies were there at the start of the experiment?

TOPIC: SEPARABLE DIFFERENTIAL EQUATIONS

PRACTICE

The scent of a certain air freshener evaporates at a rate proportional to the amount of the air freshener present. Half of the air freshener evaporates within 2 hours of being sprayed. If the scent of the air freshener is undetectable once 80% has evaporated, how long will the scent of the air freshener last?

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PRACTICE

$2,000 is invested in an account that earns interest at a rate of 8.5% and is compounded continuously. Find the particular solution that describes the growth of this account in dollars A after t years. *Hint: When interest is compounded continuously, it grows exponentially with a growth constant equivalent to the interest rate.*

TOPIC: SEPARABLE DIFFERENTIAL EQUATIONS

Newton's Law of Cooling

◆ The rate of change of an object's temp T is proportional to the diff. between T & temp of the surrounding medium T_s.

EXAMPLE

A boiled potato is 200°F before being placed on a plate in a room that is constantly 68°F. After 10 minutes, the potato's temperature is 140°F.

($\boldsymbol{A}$) Solve the initial-value problem to find $T(t)$.

HOW TO: Solve Newton's Law of Cooling Problems

1) Set up DE: plug T_s into

New

$$\frac{dT}{dt} = k(T - T_s)$$

(Newton's Law of Cooling)

2) Separate variables & integrate both sides

3) Find ___ by plugging in initial condition

(temp of object when $t = 0$)

4) Find ___ by plugging in other given temp of object

5) Solve for T

($\boldsymbol{B}$) What is the temperature of the potato after 30 minutes?

TOPIC: SEPARABLE DIFFERENTIAL EQUATIONS

EXAMPLE

A glass vase has just been formed and is removed from a furnace where it was heated to a temperature of 1250°F. It is placed in a room where the air is a constant temperature of 74°F. After 10 minutes, the temperature has dropped to 900°F. Assuming the glass cools according to the Newton's Law of Cooling, how long will it take for the vase to cool to 200°F?

TOPIC: SEPARABLE DIFFERENTIAL EQUATIONS

PRACTICE

A pie is removed from an oven and its temperature is 175°C and placed into a refrigerator whose temperature is constantly 3°C. After 1 hour in the refrigerator, the pie is 90°C. What is the temperature of the pie 4 hours after being placed in the refrigerator?