TOPIC: FINDING GLOBAL EXTREMA

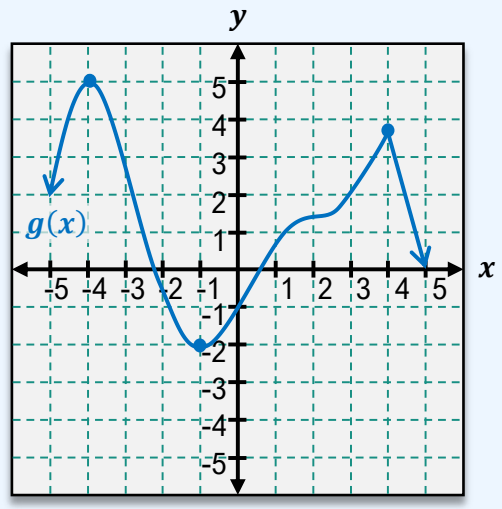
Critical Points

◆ Graphically, all local (relative) extrema either have a _____ tangent OR _____ tangent.

► This means local extrema occur at **critical points**: where $f'(x) =$ _____ OR $f'(x)$ _____.

EXAMPLE Find all the critical points of $g(x)$ & $f(x)$.

Recall **Local Extrema**



New **Critical Points**

$$f(x) = x^3 - 12x + 5$$

$f'(x) = 0$:

$f'(x)$ DNE (for x in domain):

Note: Not all critical points are local extrema.

◆ Critical points will be used to find many different characteristics of a function, including local & global extrema.

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EXAMPLE Find the critical points of the given function $f(x)$.

$$f(x) = \frac{x^2}{x-4}$$

PRACTICE Find the critical points of the given function.

(***A***)

$$g(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 12x$$

(***B***)

$$f(t) = \frac{6t}{t^2+1}$$

(***C***)

$$f(x) = \sqrt{2-x^2}$$

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EXAMPLE Find the critical points of the given function..

$$f(\theta) = 2\sin\theta + \cos^2\theta$$

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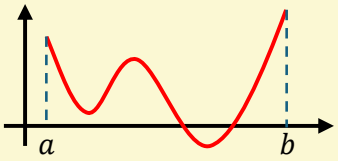
Finding Global Extrema (Extreme Value Theorem)

◆ The Extreme Value Theorem tells us how to determine if a function has a global maximum and minimum.

► If a function does have global extrema, find those values by testing the _____ & _____ points.

Extreme Value Theorem

If f is _____ on a _____ interval $[a, b]$, then f has a _____ maximum & minimum value within that interval.



EXAMPLE Given the function $f(x) = 3x^2 + 1$ answer the following.

$(\boldsymbol{A})$ Does $f(x)$ have a global max & min within the interval $[-2, 4]$?

$(\boldsymbol{B})$ If yes, find the global extrema of $f(x)$ on the interval $[-2, 4]$.

HOW TO: Find Global Extrema on a Closed Interval

1) Find critical points:

$f'(x) = 0$ or $f'(x)$ DNE

2) Plug critical pts (if in _____) & endpts into ____

3) Of values found in **(2)**:

Largest = global _____

Smallest = global _____

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EXAMPLE Find the absolute maximum and minimum values of the function on the given interval.

$f(x) = 2x^4 - 8x^3 - 16x^2 + 3; [-2, 5]$

HOW TO: Find Global Extrema on a Closed Interval

1) Find critical points:
$f'(x) = 0$ or $f'(x)$ DNE

2) Plug critical pts (if in **interval**) & endpts into $\boldsymbol{f(x)}$

3) Of values found in **(2)**:
Largest = global **MAX**
Smallest = global **MIN**

PRACTICE Find the global maximum and minimum values of the function on the given interval. State as ordered pairs.

(A)

$y = x + \frac{2}{x}; [0.25, 3]$

(B)

$y = 8 + 27x - x^3; [0, 4]$

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EXAMPLE Find the global maximum and minimum value(s) of the function over $(-\infty, \infty)$ or state that there isn't one.

$y = x^2 + 6x - 3$

HOW TO: Find Global Extrema on a Closed Interval

1) Find critical points:

$f'(x) = 0$ or $f'(x)$ DNE

2) Plug critical pts (if in **interval**) & endpts into $\boldsymbol{f(x)}$

3) Of values found in **(2)**:

Largest = global **MAX**

Smallest = global **MIN**

EXAMPLE Find the absolute maximum and minimum values of the function on the given interval.

$f(x) = x + \cos x\,; [0, 2\pi]$